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Du micromètre à l'attomètre : une détermination optique de la taille du proton

François Nez

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Du micromètre à l'attomètre : une détermination optique de la taille du proton

F.D Amaro, A. Antognini, F. Biraben, J.M.R. Cardoso, D.S. Covita, A. Dax, S. Dhawan, L.M.P. Fernandes, A. Giesen, T. Graf, T.W. Hänsch, P. Indelicato, L. Julien, C.-Y. Kao, P.E. Knowles, F. Kottmann, J.A.M. Lopes, E. Le Bigot, Y.-W. Liu, L. Ludhova, C.M.B. Monteiro, F. Mulhauser, T. Nebel, F. Nez, R. Pohl, P. Rabinowitz, J.M.F. dos Santos, L.A. Schaller, K. Schuhmann, C. Schwob, D. Taqqu, J.F.C.A. Veloso



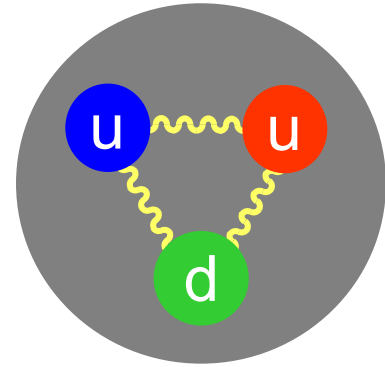
<http://muhy.web.psi.ch>

<http://www.lkb.ens.fr/-Metrologie-Quantique->

Proton

Structure

2 quarks **up** ($2/3 e$) + 1 quark **down** ($-1/3 e$) + strong interaction (**gluons**)



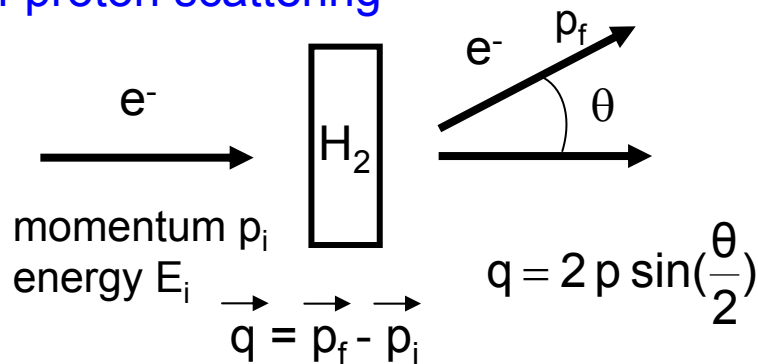
“proton size” $\lambda_c = \frac{h}{m_p c} \approx 1.3 \text{ fm}$

Proton charge radius : $r_p \sim 0.8 \text{ fm}$

$$r_p = \sqrt{\langle r^2 \rangle} \quad \langle r^2 \rangle = \frac{\int r^2 \rho(r) d^3r}{\int \rho(r) d^3r}$$

$\rho(r)$: proton charge distribution

Electron-proton scattering

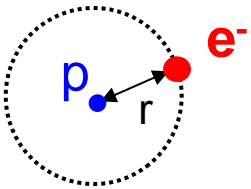


$$\frac{d\sigma}{d\Omega}(E_i, \theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i, \theta) \right]}_{\text{Rutherford}} G(q^2)$$

$$G(q^2) = \int d^3r e^{iqr} \frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6} q^2$$

r_p deduced from extrapolation $q \rightarrow 0$ with $\rho(r)$ unknown...

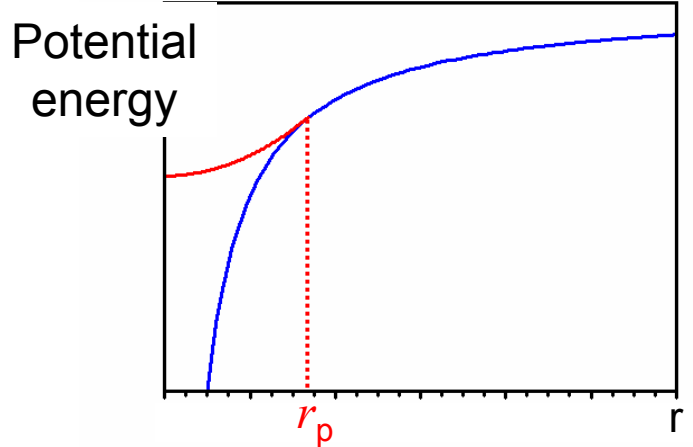
hydrogen spectroscopy



$$E(n,l,j) = \underbrace{\text{Dirac} + \text{recoil}}_{\text{exact}} + \underbrace{L(n,j)}_{\text{not well known}}$$

$$hcR_\infty f(\alpha, m_e/m_p, n,l,j)$$

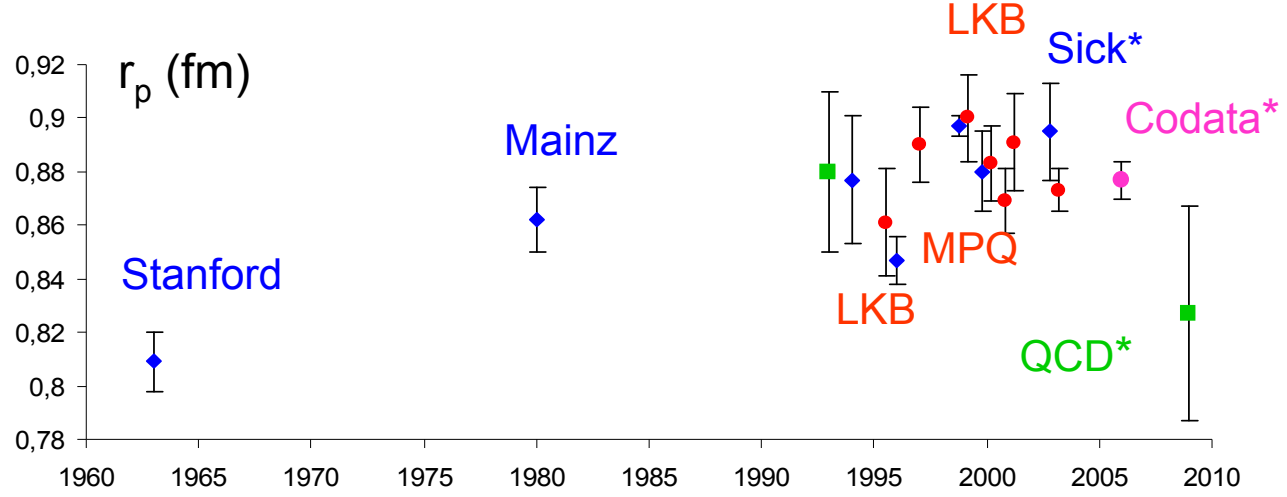
- QED corrections ($1/n^3$)
- relativistic recoil
- charge radius of the proton ($1/n^3$)



$$f_{1S-2S} = 2\,466\,061\,413\,187\,035 (10) \text{ Hz}$$

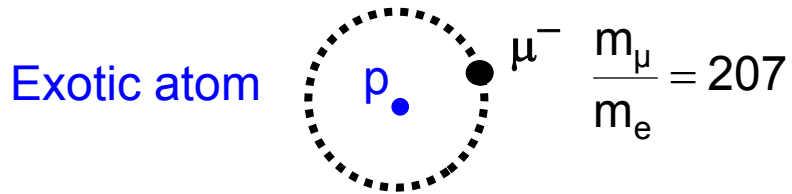
PRL 107 203001 (2011)

Previous determinations of the "proton radius"



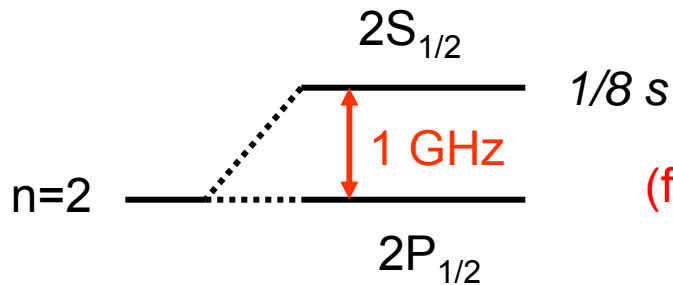
* Hydrogen data(1S-2S and 2S-nS/nD) + QED re-evaluation : 0,8760 (78) fm
 * Electron scattering (all meas+re-analysis): 0,895 (18) fm $f_{1S-2S} = 2\,466\,061\,413\,235\,502$ Hz
 * QCD (lattice cal.) : 0,827 (40) fm

Muonic hydrogen spectroscopy



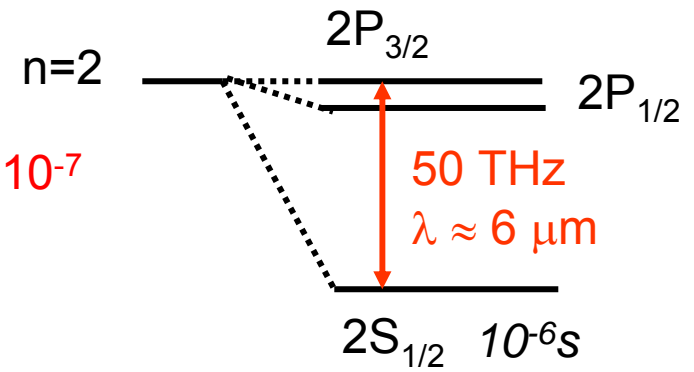
radius : $\sim a_0/200$

electronic hydrogen : e-p



$$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \approx 10^{-7}$$

muonic hydrogen : μ -p



Relative contribution
due to r_p to 2S-2P

$$1.4 \times 10^{-4}$$

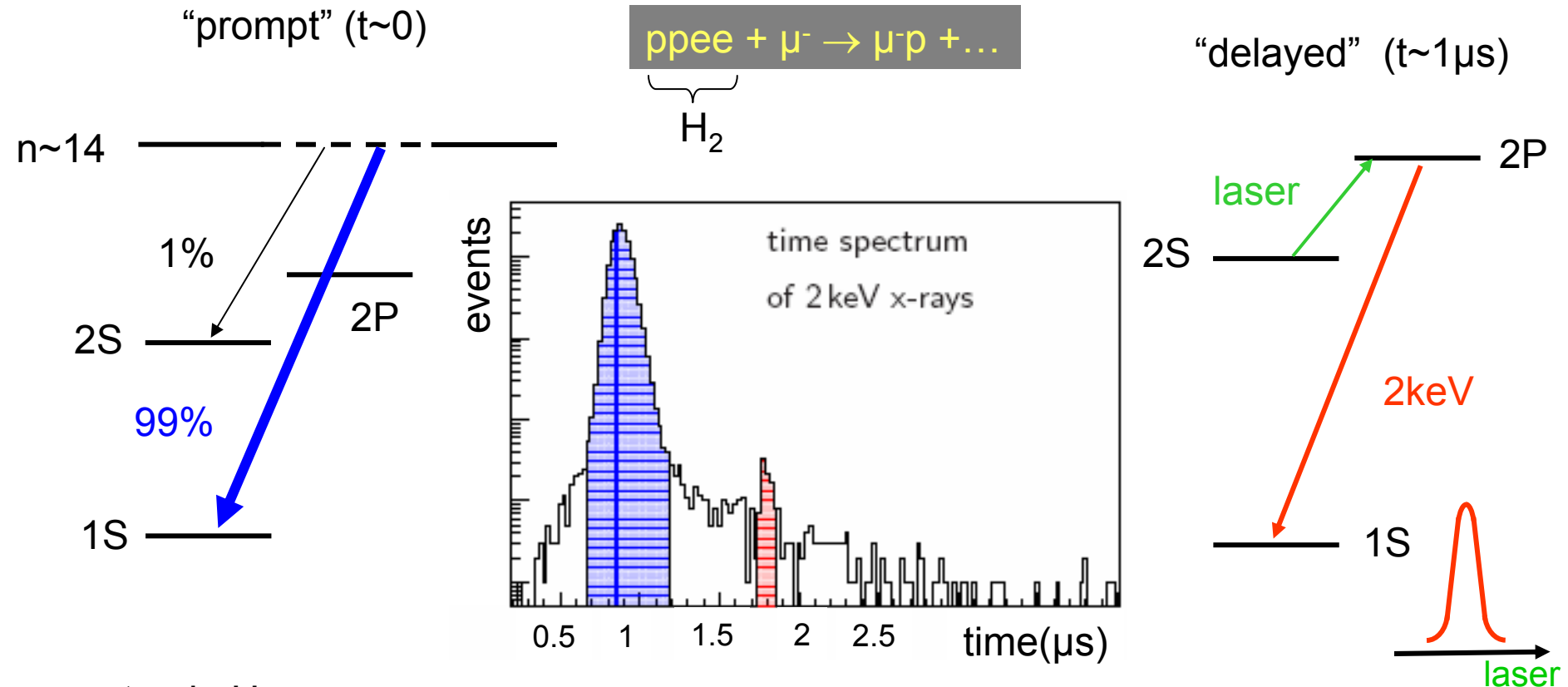
$$2 \times 10^{-2}$$

Predicted frequency of muonic hydrogen 2S-2P transition (2002)

$$\Delta E_{2S-2P} = 50.7702(12) - 1.2634 r_p^2 - 0.0084 r_p^3 = 49.8063 (149) \text{ THz}$$

$$\text{with } r_p = 0.8760(68) \text{ fm}$$

Principle of the experiment



μ^- stop in H_2 gas
 $\Rightarrow \mu p^*$ atoms formed ($n \sim 14$)

99%: cascade to 1S emitting prompt $K\alpha, K\beta, \dots$

1%: long lived 2S state ($\tau \sim 1\mu s$ at 1mbar)

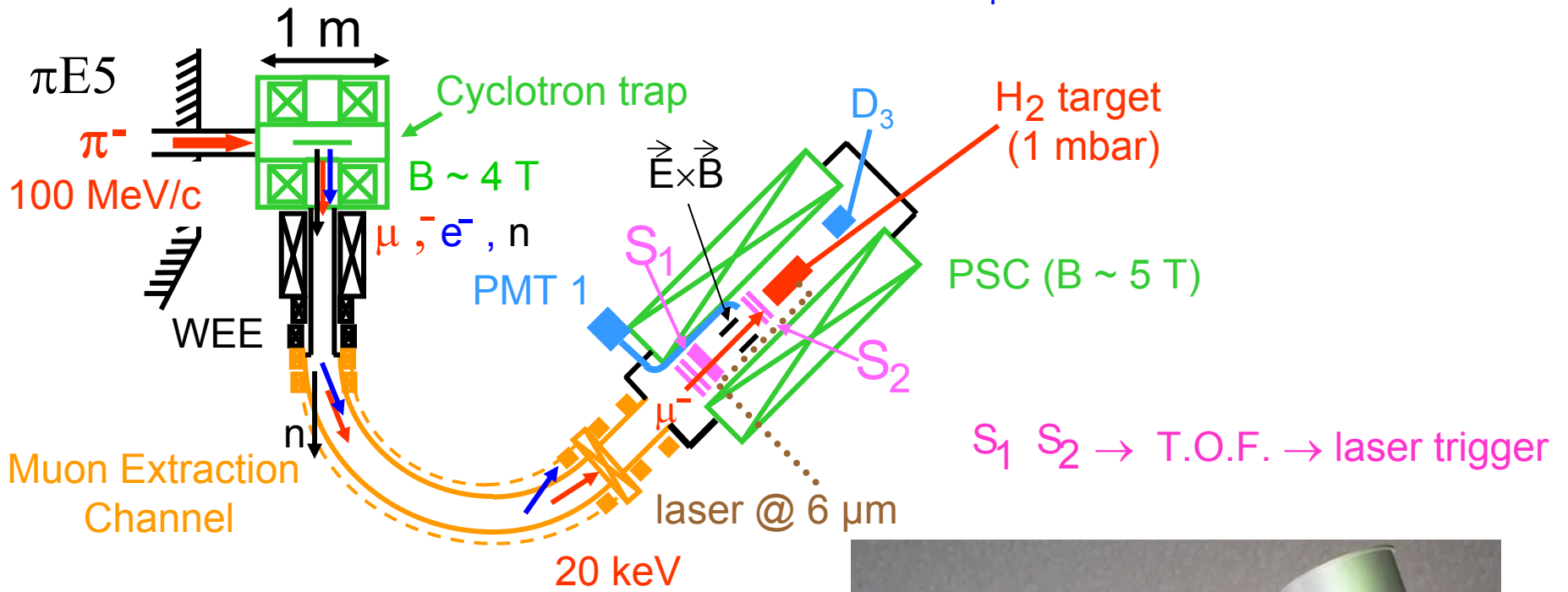
Fire laser ($\lambda \sim 6\mu m, \Delta E \sim 0.2 eV$)
 \Rightarrow induce $\mu p(2S-2P)$

\Rightarrow observe delayed $K\alpha$ x-rays

\Rightarrow normalize $\frac{\text{delayed } K\alpha}{\text{prompt } K\alpha}$ x-rays

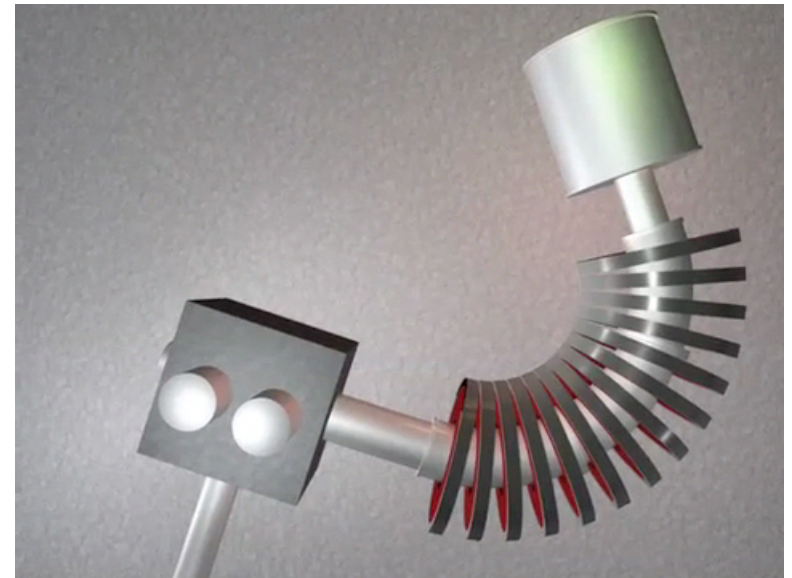
Muonic hydrogen source

p (500 MeV, 2 mA) \rightarrow carbon target $\rightarrow \pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ (300 μ /s)

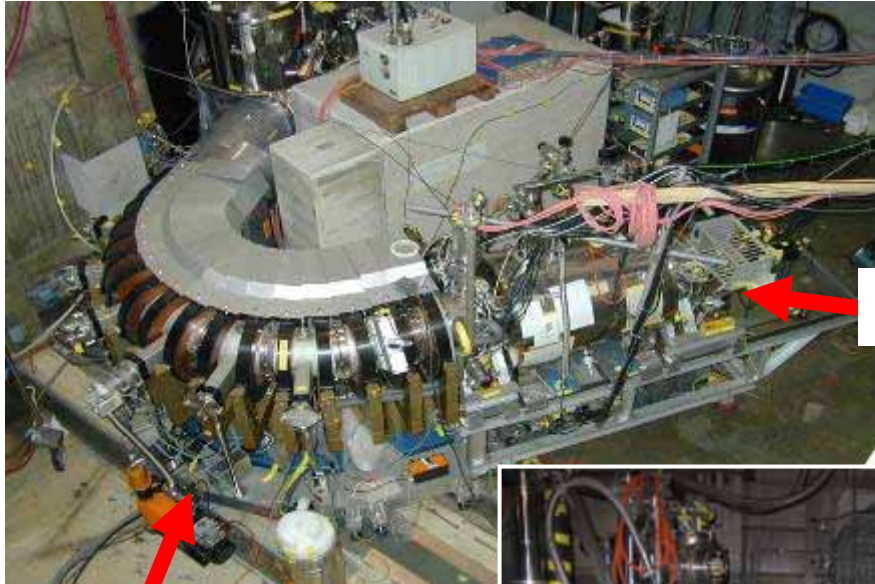


muon life in CREMA experiment

- pion π^-
- muon μ^-
- photon from scintillator



muon beam apparatus

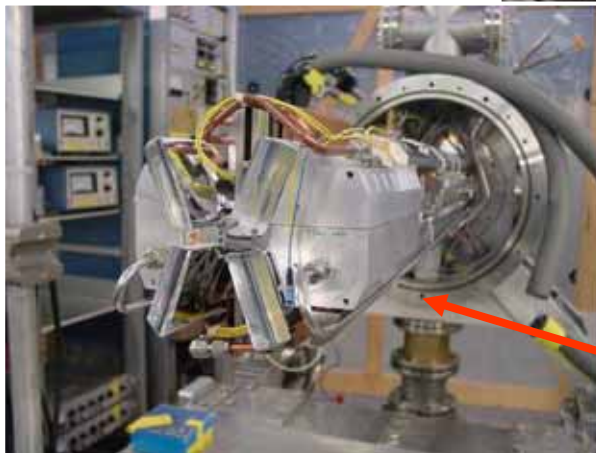


PSC solenoid,

Muon extraction channel

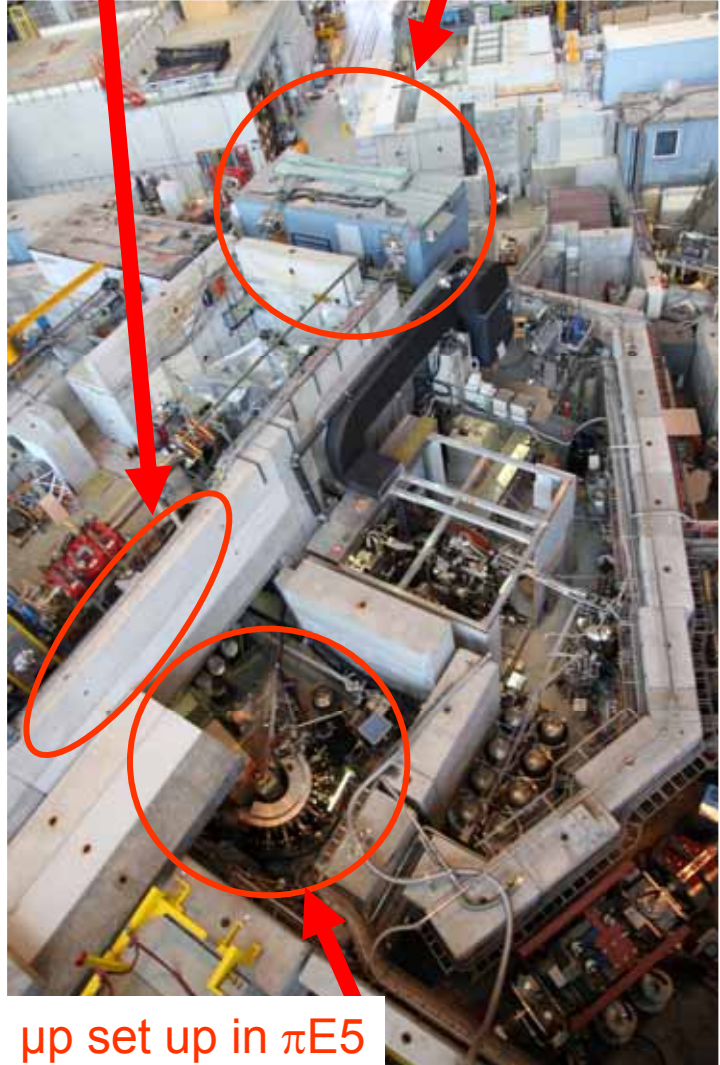


H2 target, laser cavity, detectors



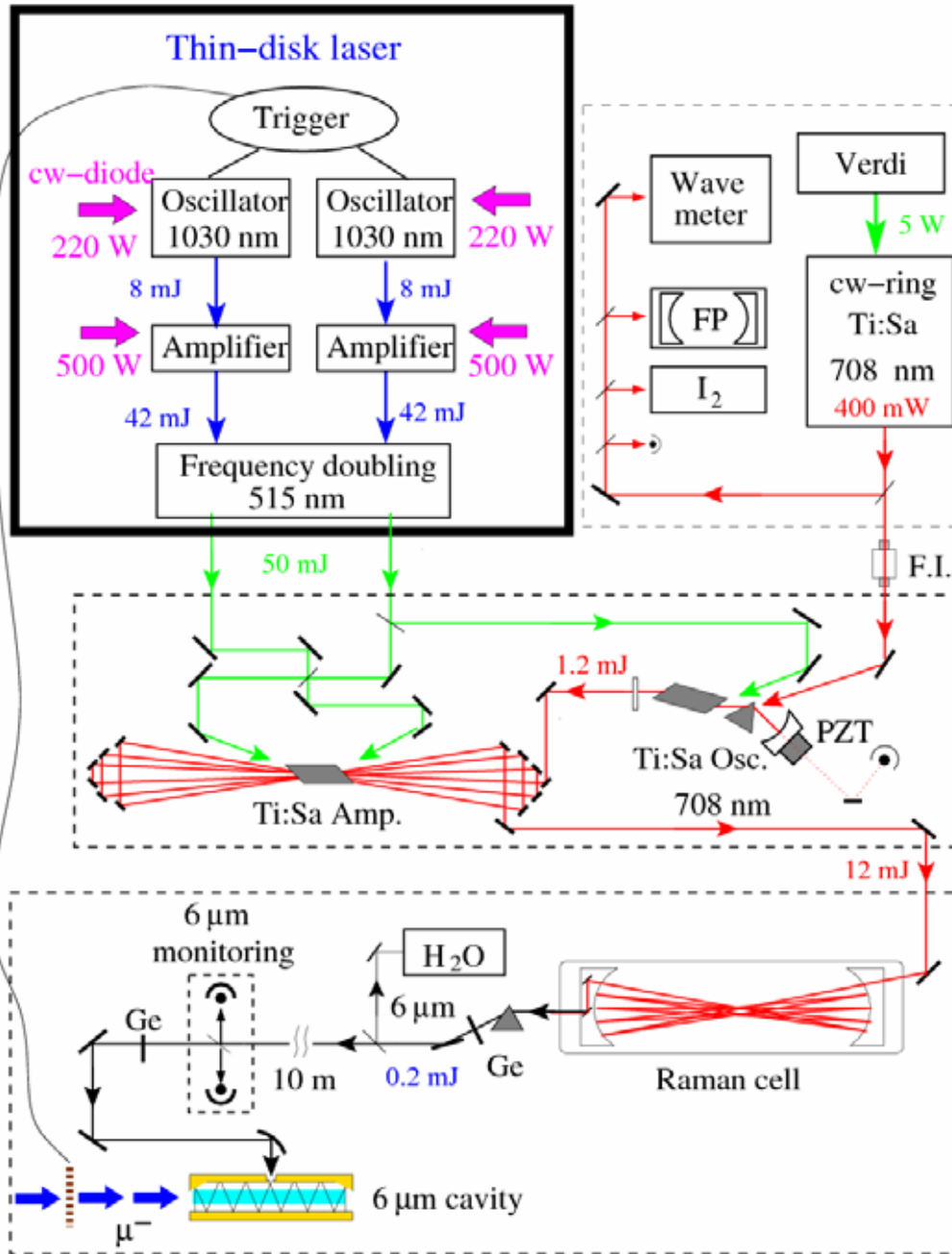
laser hut
below
concrete blocks

counting room



μp set up in $\pi E5$

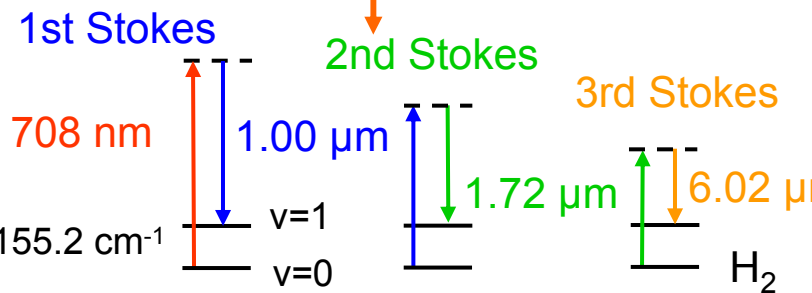
Laser chain



- Each single muon triggers the laser system (random trigger)
- 2S lifetime $\sim 1 \mu\text{s}$ \rightarrow **short laser delay (disk laser)**
- **6 μm tunable** laser pulse (0.2 mJ)

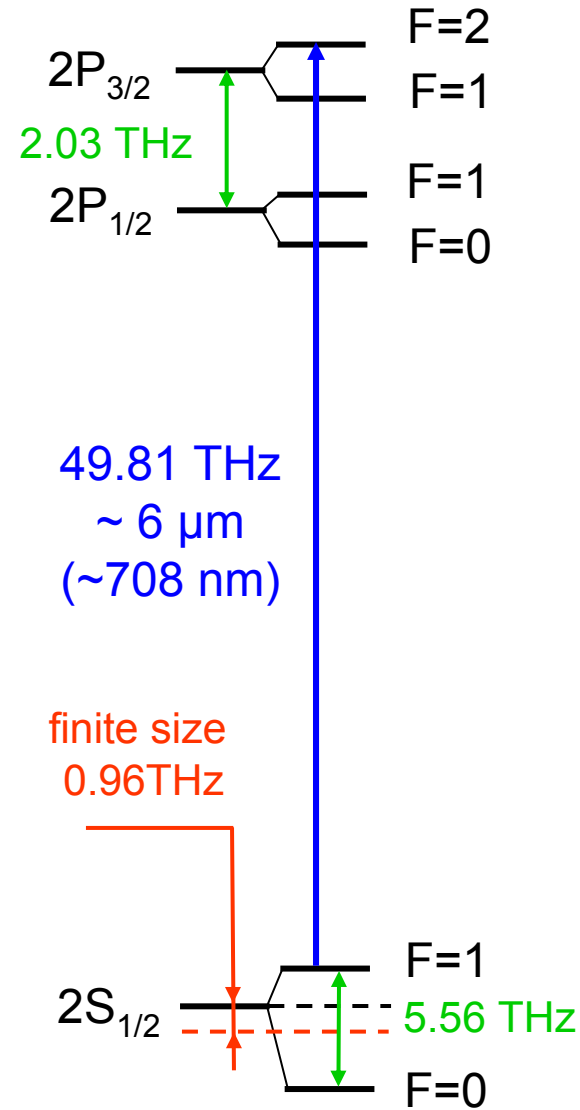
μ^- trigger
 ↓
 Thin disk laser (1030 nm) + LBO (515 nm)

↓
 pulsed TiSa oscillator + amplifier
 (708 nm determined by cw TiSa seeding)

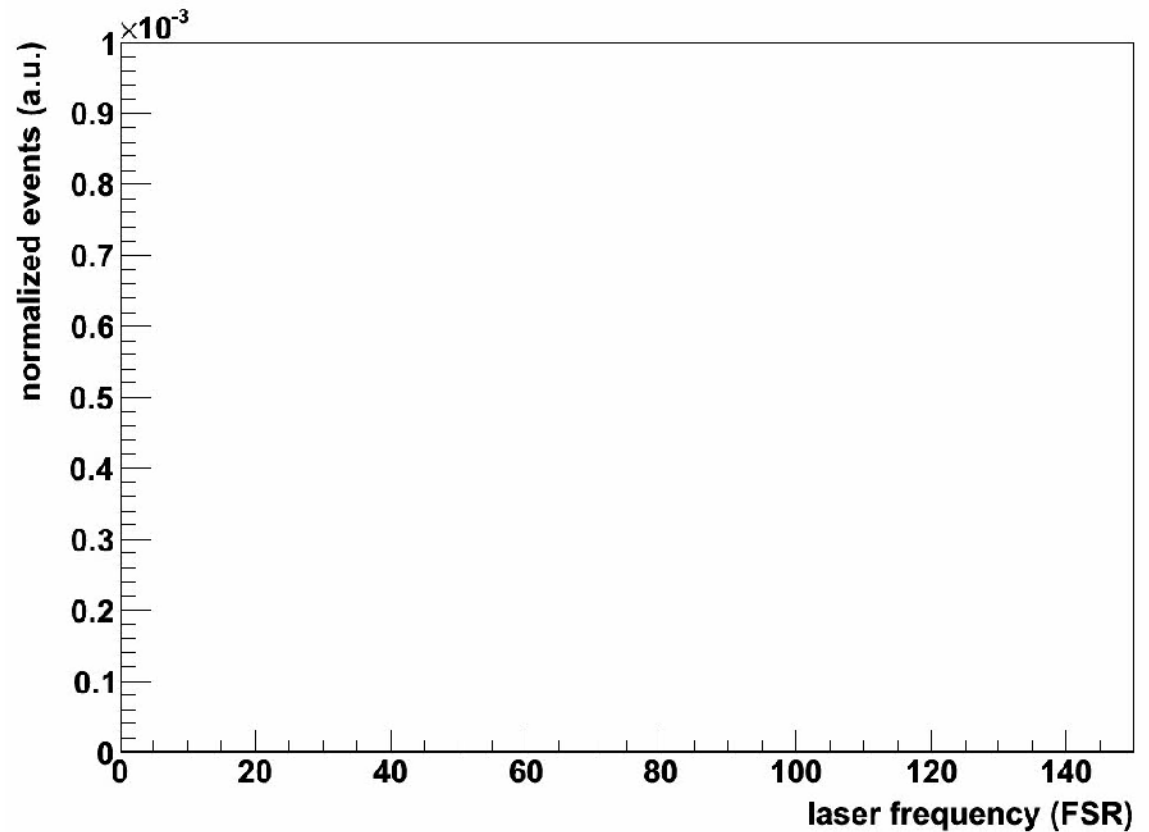


↓
 Multipass cavity at 6 μm
 surrounding the H_2 target

muonic hydrogen : $2S_{1/2}(F=1) - 2P_{3/2}(F=2)$



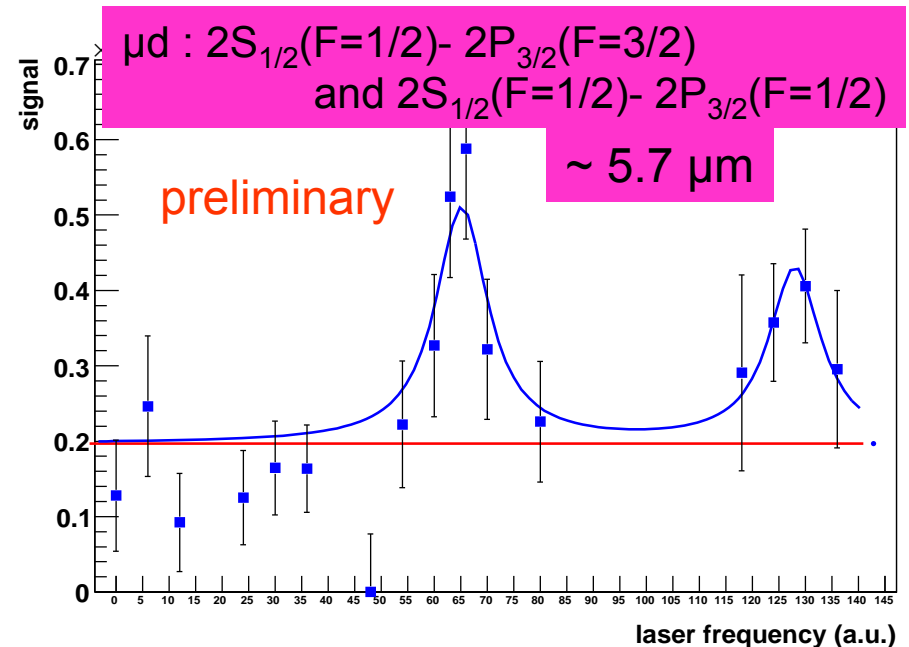
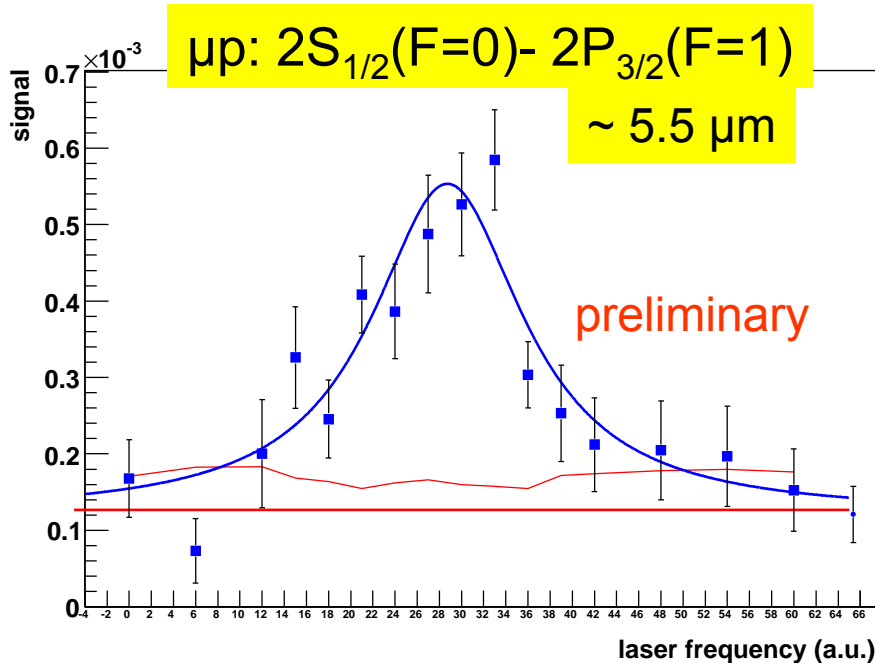
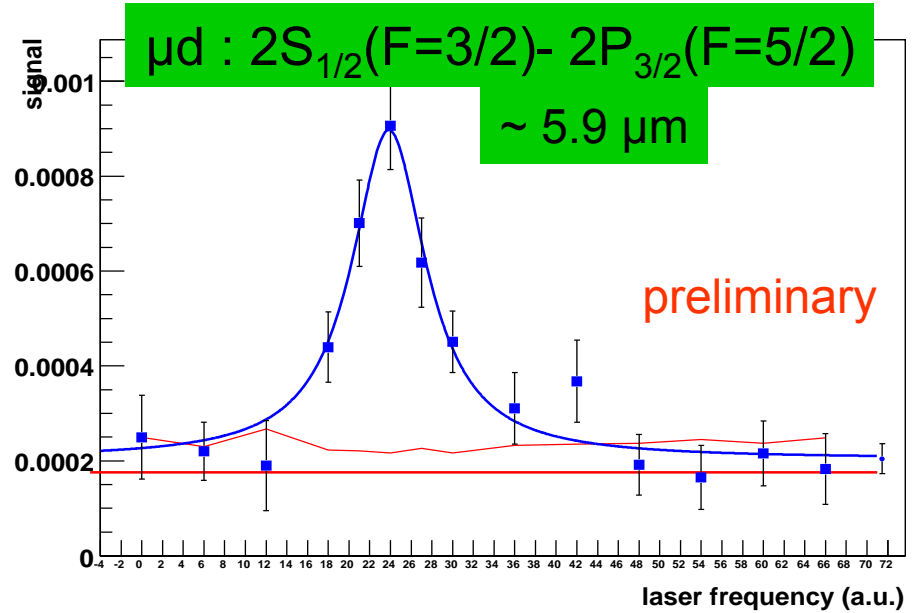
- 550 events measured
- 155 backgrounds
- 31 FP fringes
- 250 hours



→ proton charge radius ($\sim 0.1\%$)

Other lines observed in μp and μd atoms

- **2nd μp line** : at the correct position
hfs \rightarrow **Zeemach radius**
- **μd** : not at the correct position with
new rp and isotope shift \rightarrow **deuteron polarizability**
- **2nd μd line** : observation \rightarrow **check calculation**



Comparison with other best determinations

$$\nu (\mu p : 2S_{1/2}(F=1) - 2P_{3/2}(F=2)) = 49\,881.88 \text{ (76) GHz}$$

$$\nu_{\text{theo}} = 50772.43 \text{ (118)} - 1263.69 r_p^2 + 8.39 r_p^3 \text{ GHz}$$

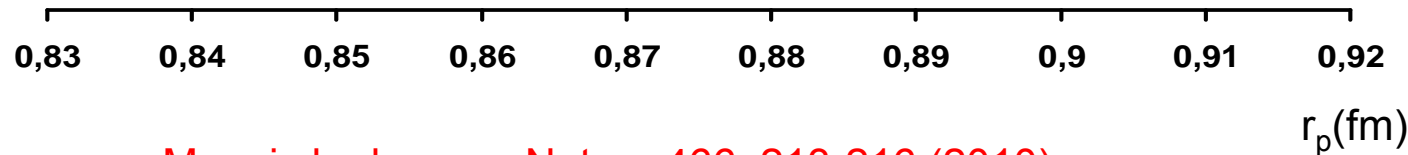
$$r_p = 0.84184 \text{ (36)(56) fm}$$

H/D spectroscopy + QED ● 0.8760 (78) fm

$\mu p (2S_{1/2} - 2P_{3/2}) + \text{QED}$ ● 0.84184 (68) fm

New electron-proton scattering ● 0.879 (8) fm

electron-proton scattering re-analysis ● 0,895 (18) fm



Muonic hydrogen : Nature 466, 213-216 (2010)

New elec-prot scattering : Phys Rev Lett 105, 242001 (2010)

$$f_{1S-2S} = 2\,466\,061\,413\,187\,035 \text{ (10) Hz} \quad \text{PRL 107 203001 (2011)}$$

$$f_{1S-2S} = 2\,466\,061\,413\,109\,090 \text{ Hz}$$



New r_p (μp ($2S_{1/2}(F=1) - 2P_{3/2}(F=2)$)) : outcomes

Muonic hydrogen Lamb shift theory ? checked

Hydrogen spectroscopy, Rydberg constant R_∞ ?

Codata 2006 “The experimental and theoretical H and D data remain highly consistent.”

First measurement of $1S-3S$ (LKB) : 2 922 743 278 678(13) kHz. EPJ D **60** p.243 (2010)
Prediction : 2 922 743 278 671.6(1.4) kHz PRL **95**, 163003 (2005)

“New physics” ?

“We consider exotic particles that, among leptons, couple preferentially to muons and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1, and spin-2 particles as an explanation.” PRL **106**, 153001 (2011)

Example of constrain : magnetic moment anomaly of the electron (Harvard) $g-2 = f(\alpha/\pi)$

With recoil (LKB) $v_r = \frac{\hbar k}{m(\text{Rb})}$ $\alpha^2 = \frac{2R_\infty}{c} \frac{m(\text{Rb})}{m_p} \frac{m_p}{m_e} \frac{h}{m(\text{Rb})}$ PRL **106**, 080801 (2011)

Polarizability of the proton ?

Model independent analysis of proton structure...: Phys Rev Lett 107, 160402 (2011)

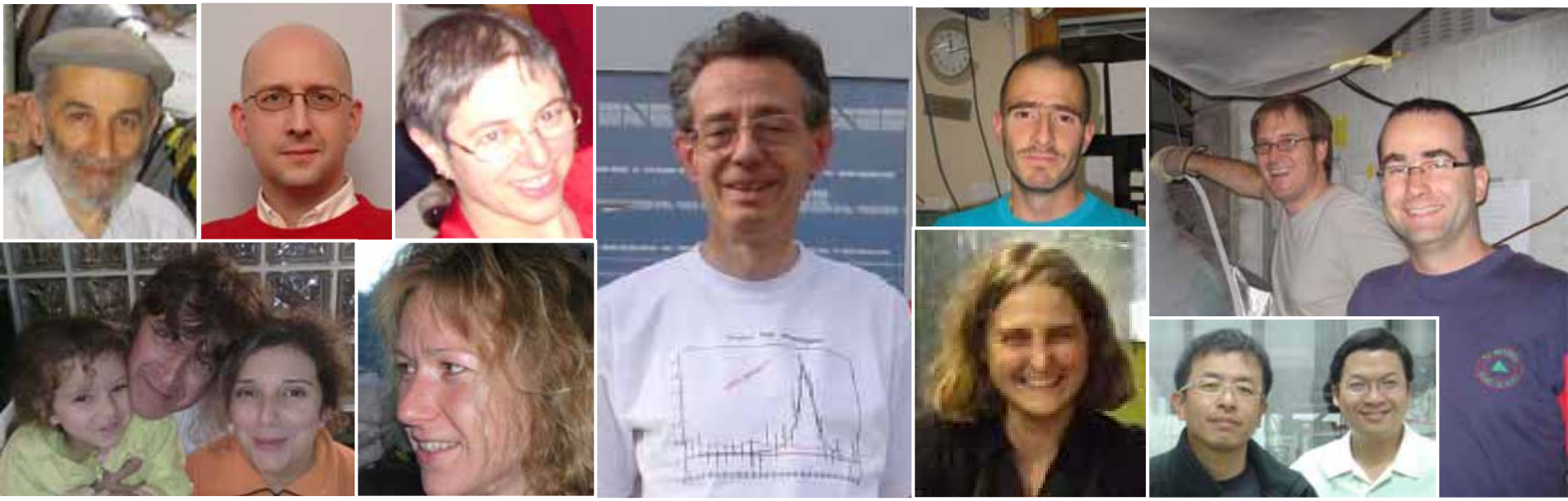
$$\left. \begin{aligned} v(\mu p : 2S_{1/2}(F=1) - 2P_{3/2}(F=2)) &= 49\,881.88 \text{ (76) GHz} \\ v_{\text{theo}} &= 50772.43 \text{ (967)} - 1263.69 r_p^2 + 8.39 r_p^3 \text{ GHz} \end{aligned} \right\} r_p = 0.84184(36)(458) \text{ fm}$$

Hydrogen spectroscopy, QED tests, ...

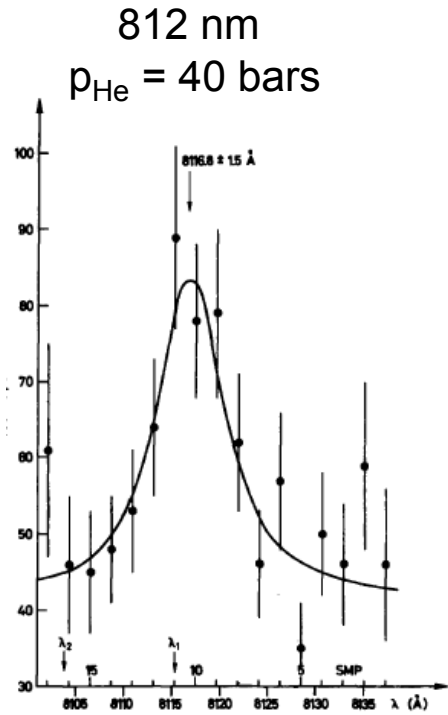
- Hydrogen Deuterium 2S-nS/nD spectroscopy at NPL,
J.L. Flowers et al IEEE Trans. Instrum. Meas. 56, (2007) 331.
- $^{20}\text{Ne}^{9+}$ Rydberg states at NIST-Gaithersburg , U. D. Jentschura et al, PRL 100, 160404 (2008).
- Hydrogen 1S-2S at MPQ-Garching hydrogen cooled with magnetic field
- Hydrogen 1S-3S picosecond spectroscopy at MPQ-Garching E. Peters on going PhD
- Hydrogen 1S-3S cw-spectroscopy at LKB-Paris S.Galtier on going PhD
- He^+ spectroscopy femtosecond spectroscopy at MPQ-Garching
M. Herrmann et al, PRA 79, 052505 2009.
- Muonic helium ($\mu^4\text{He}^+$) spectroscopy at PSI-Villigen CREMA collaboration Proposal 2010.
- Electron-proton scattering MAMI/A1 collaboration at Mainz
J. C. Bernauer et al., Can. J. Phys. 85, 419 (2007).
- QED test : a determination from recoil at LKB-Paris **Looking for PhD student**
- QED numerical calculation...
- QCD lattice calculation...



Proton Size Investigators thank you for your attention



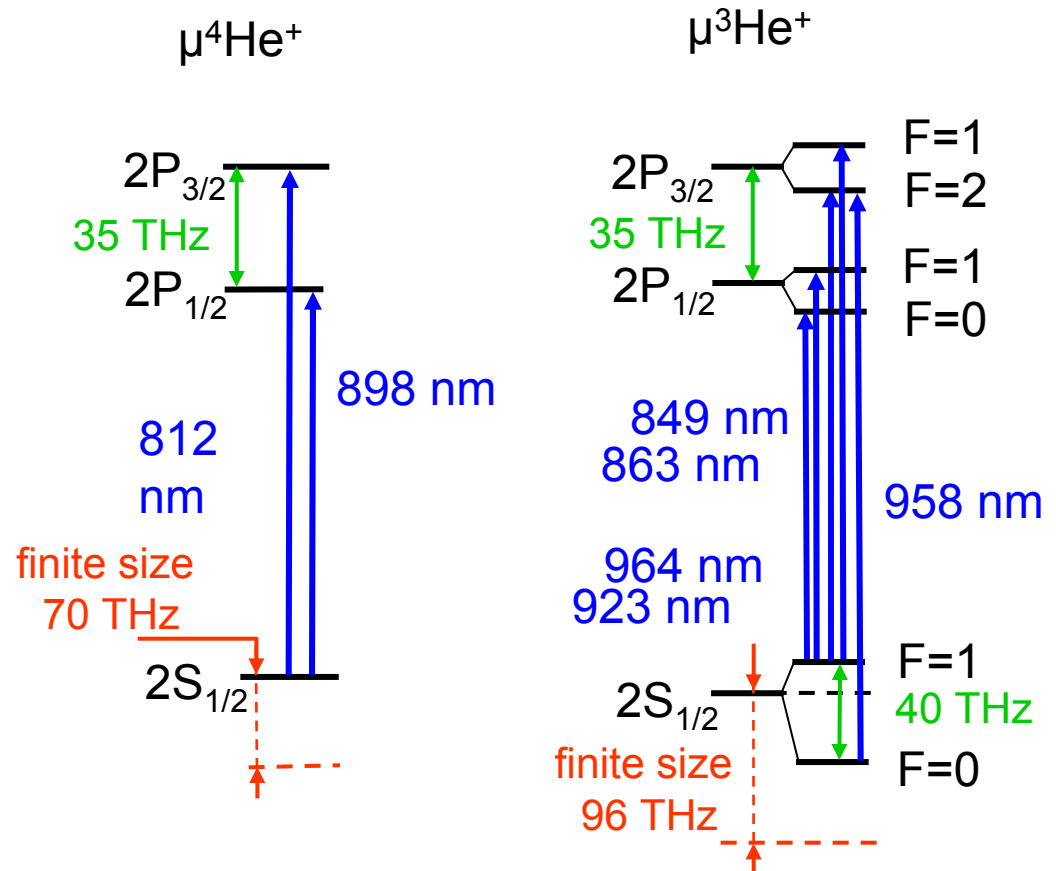
Prospect : muonic helium spectroscopy at PSI



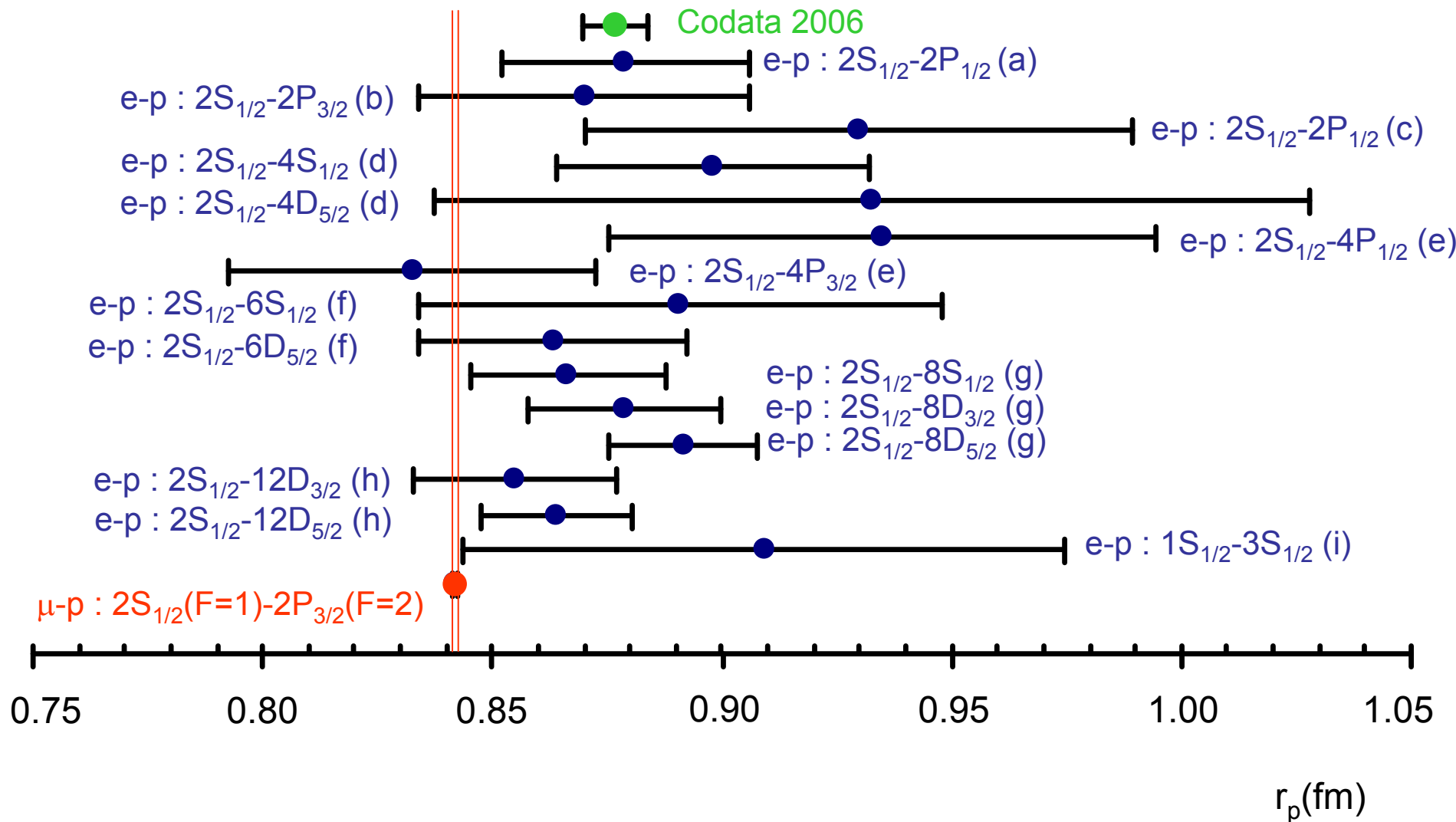
Nuclear Physics A278 (1977) p. 381

but signal never reproduced
 (10 bars, 40 bars)

2011-2013 → muonic helium spectroscopy (4 mbar)



- μHe^+ spectroscopy + He^+ spectroscopy → QED test ($Z\alpha$)
- test of the spectroscopy of muonic system



New r_p (μp ($2S_{1/2}(F=1) - 2P_{3/2}(F=2)$)) : outcomes

“New physics”:

Constrains due to other high precision determination (S.Karshenboim PRL104, 220406 (2010), U.D. Jentschura, Annals of Physics (2010), V. Barger et al, arXiv:10113519V1, ...)

“We consider exotic particles that among leptons, couple preferentially to muons, and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1 and spin-2 particles as an explanation.” V.Barger et al (Nov 2010)

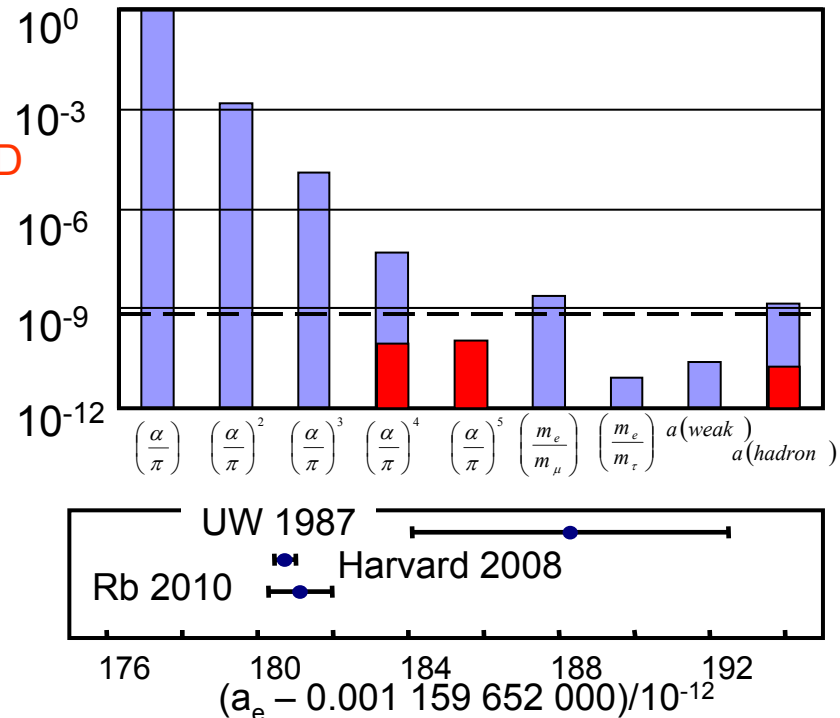
Example of constrain : magnetic moment anomaly of the electron (Gabielse, Kinoshita)

$$a_e = \frac{|\mu_e|}{\mu_B} - 1 = A_1 \left(\frac{\alpha}{\pi} \right) + \dots + A_5 \left(\frac{\alpha}{\pi} \right)^5 + a_e(\text{muon}) + a_e(\text{tau}) + a_e(\text{weak}) + a_e(\text{hadron})$$

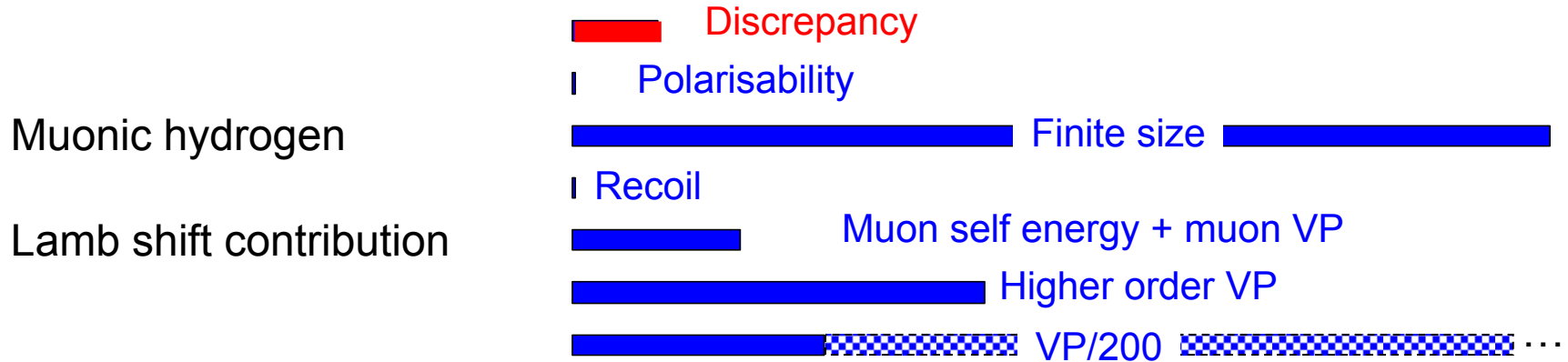
With our new determination of α : stringent test of QED

Recoil $v_r = \frac{\hbar k}{m(\text{Rb})}$ $\alpha^2 = \frac{2R_\infty}{c} \frac{m(\text{Rb})}{m_p} \frac{m_p}{m_e} \frac{h}{m(\text{Rb})}$

Paris measurement of α (Rb-2010) (6.6×10^{-10})
submitted to PRL (15 Dec 2010), arXiv 1012.3627v1



Lamb shift - form factor – polarizability

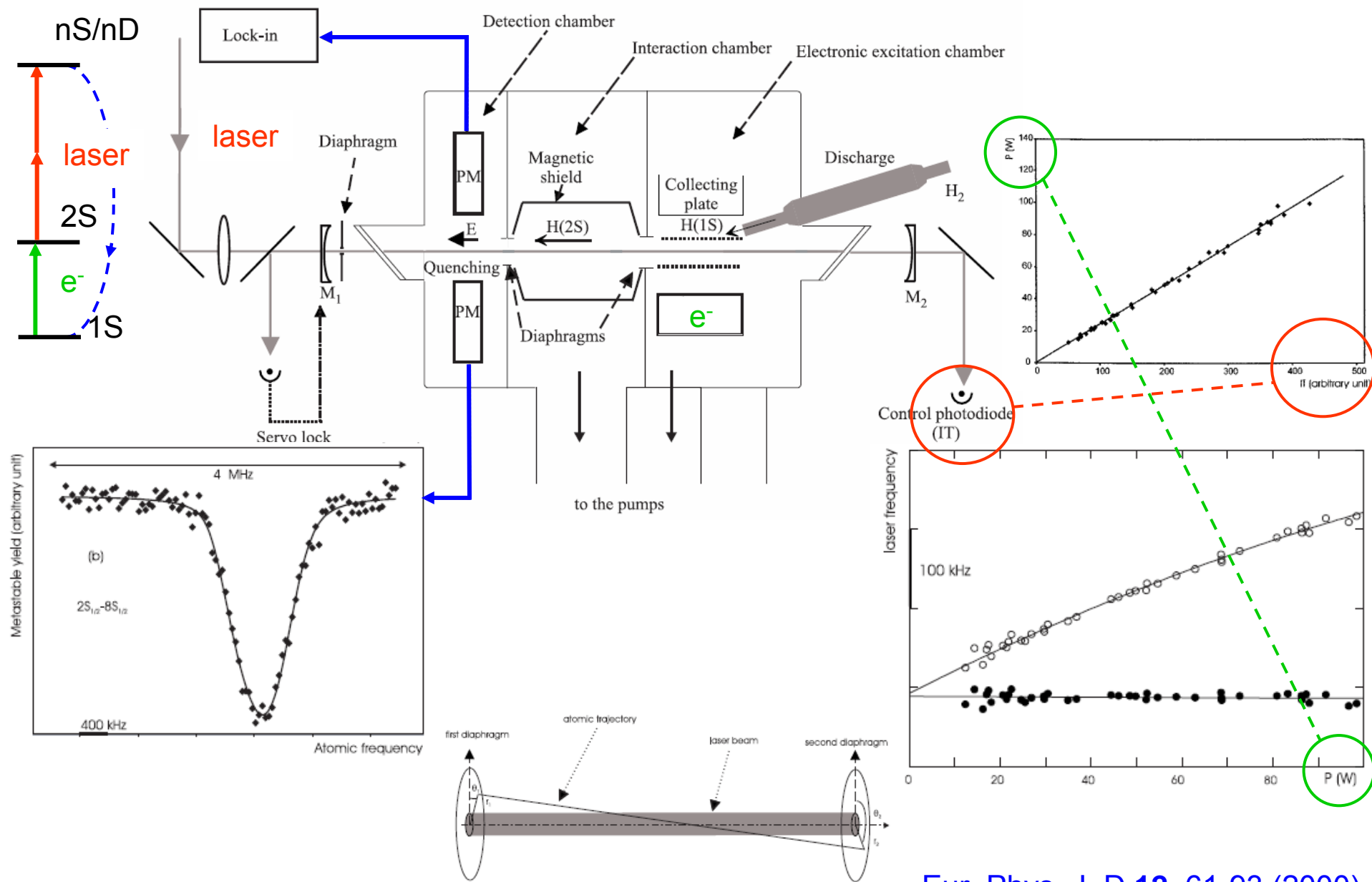


➤ Dipole form factor $\rho(r) = e^{-\frac{r}{R}}$

➤ Gaussian form factor $\rho(r) = e^{-\frac{r^2}{R^2}}$

	polarizability	form-factor
• 2S state electronic hydrogen (e-p)	8.7 Hz	
• 2S state electronic deuterium (e-d)	2.7 kHz	
• 2S state muonic hydrogen (μ -p)	3.6 GHz	500 MHz
• 2S state muonic deuterium (μ -d)	360 GHz	

2S-nS/nD hydrogen/deuterium spectroscopy

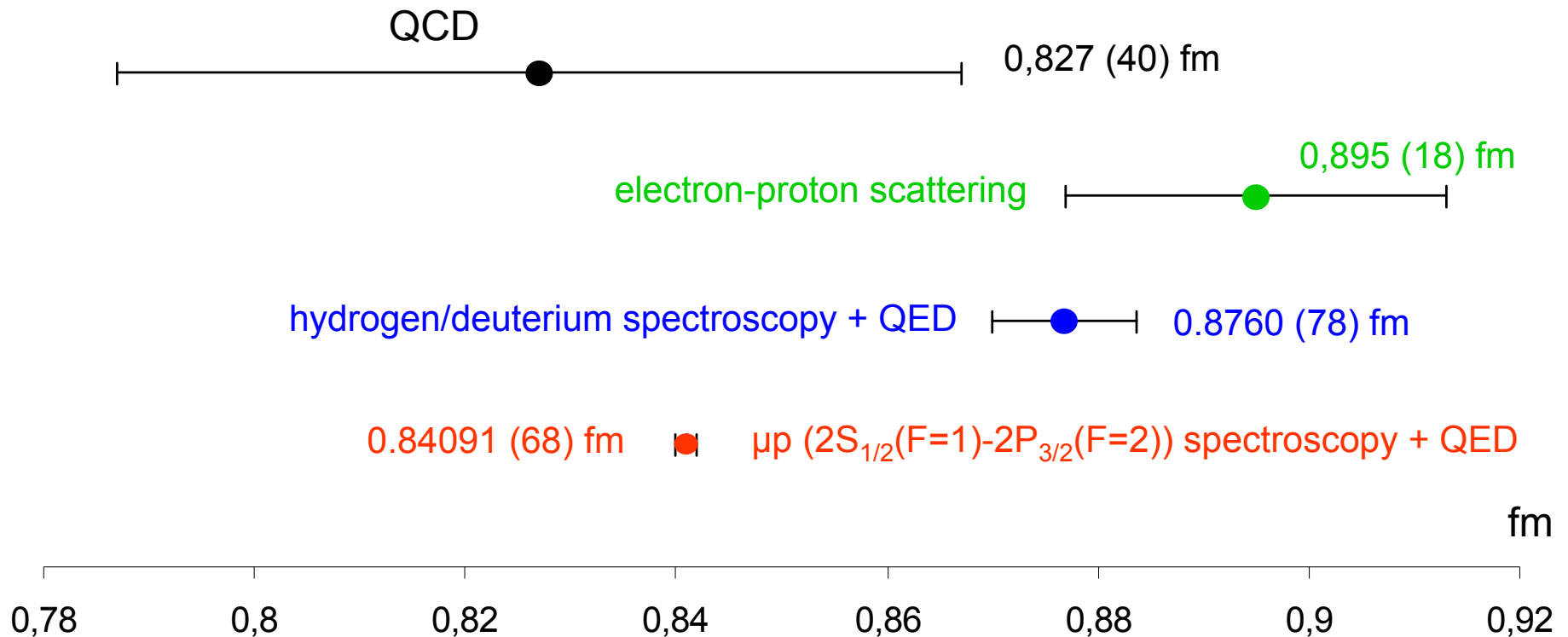


Comparison with other best determinations

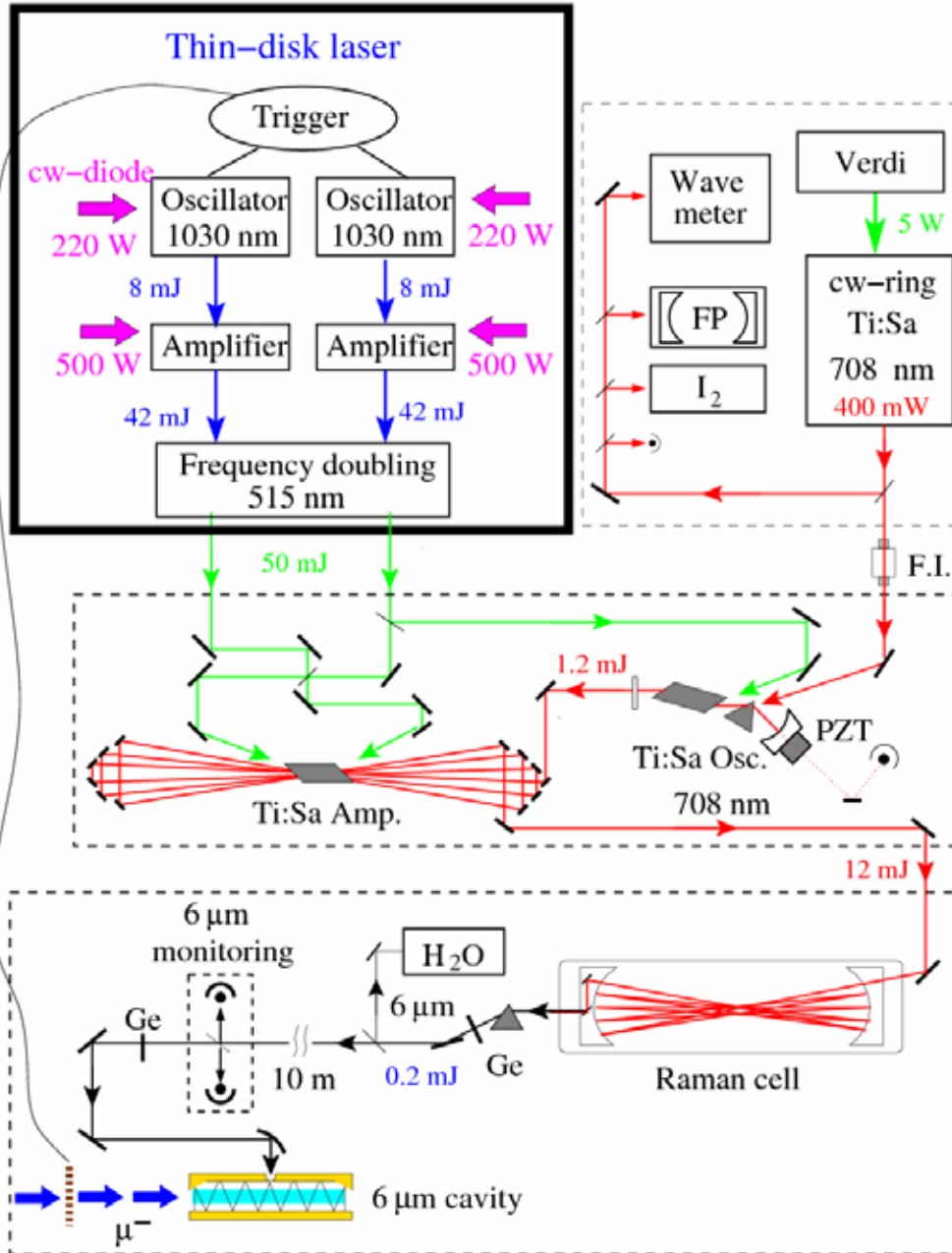
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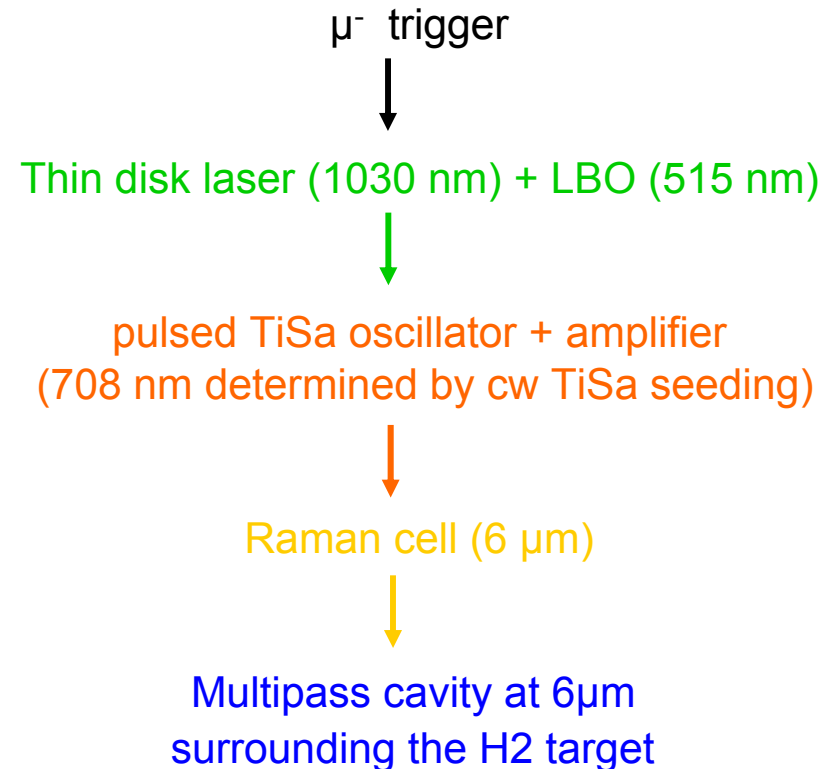
$$r_p = 0.84184 \text{ (36)(56) fm}$$



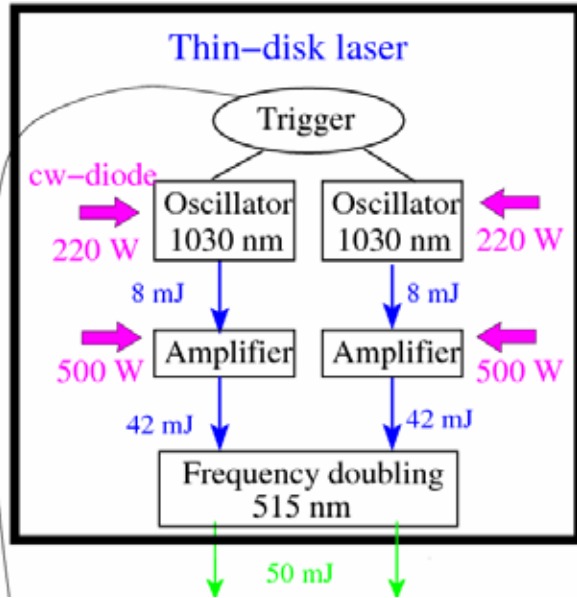
Laser chain : principle



- Each single muon triggers the laser system (random trigger)
- 2S lifetime $\sim 1\mu\text{s}$ \rightarrow short laser delay
- 6 μm tunable laser pulse (0.2mJ)

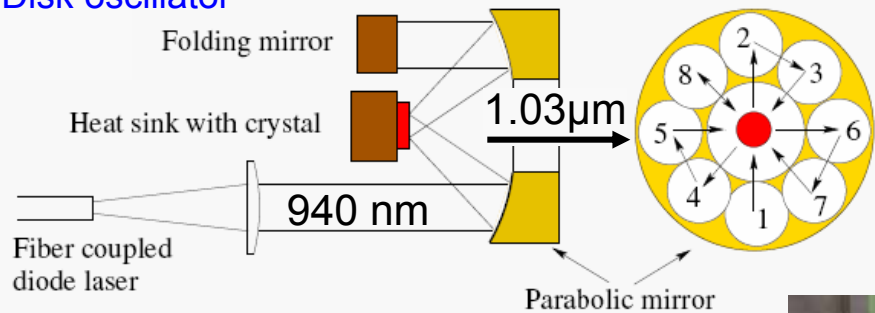


Laser chain : thin-disk Yb:YAG laser

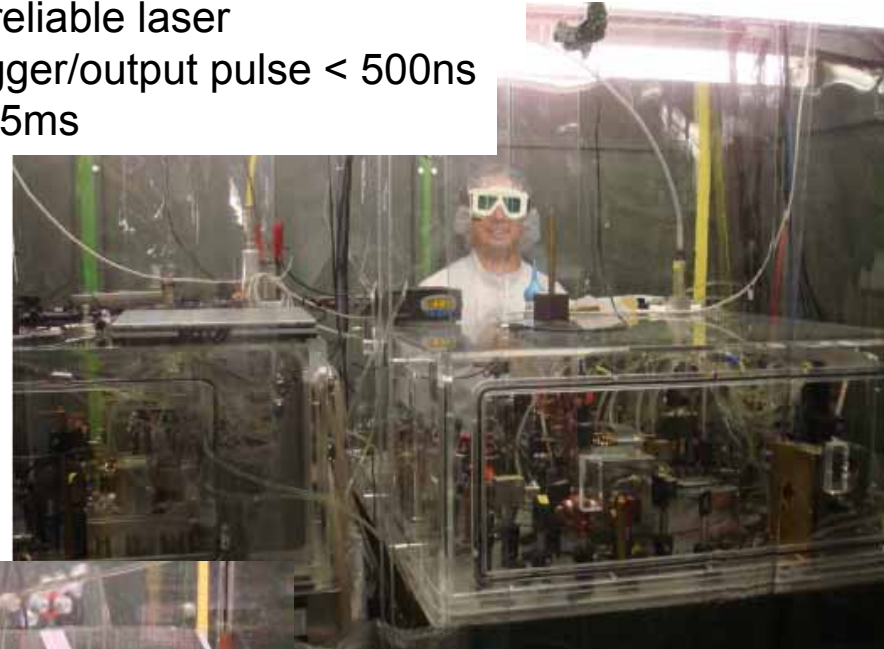
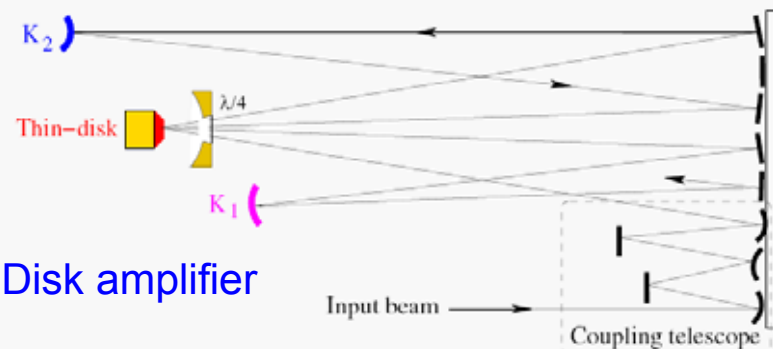


- Q-switch oscillator (cw prelasing mode → short delay between electronic trigger and opt. output (8mJ, xxx ns, delay 250ns)
 - 12 pass amplifier (20m propagation) (42mJ)
 - LBO frequency doubling 25mJ @ 515nm
- 50 mJ @ 515 nm, reliable laser
 - delay electronic trigger/output pulse < 500ns
 - random trigger ≥ 1.5ms

Disk oscillator



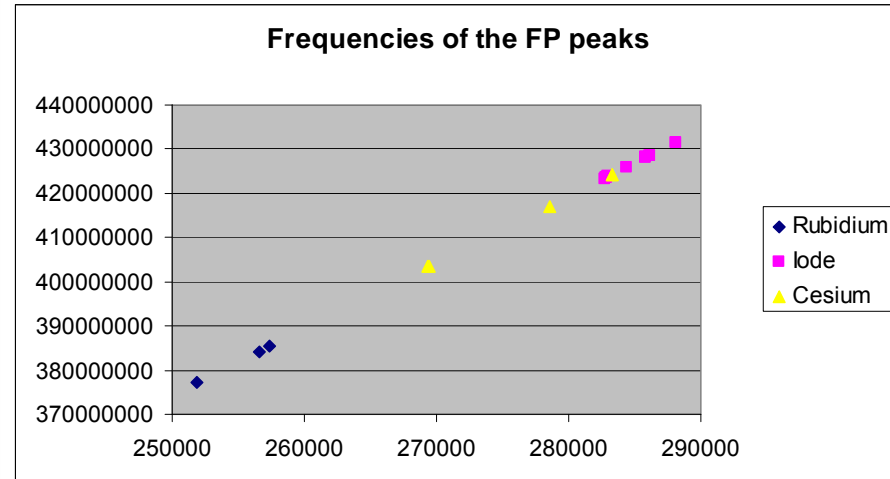
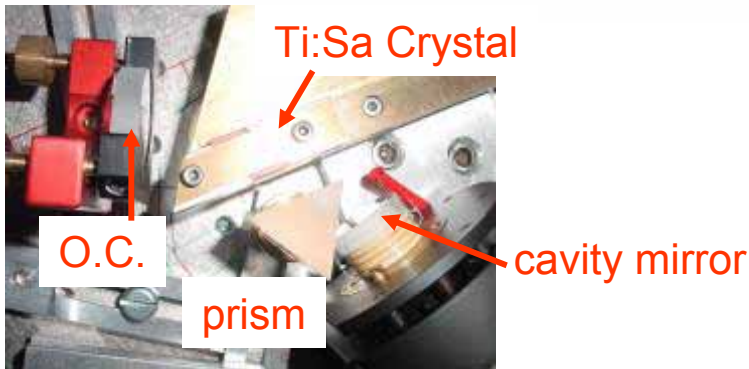
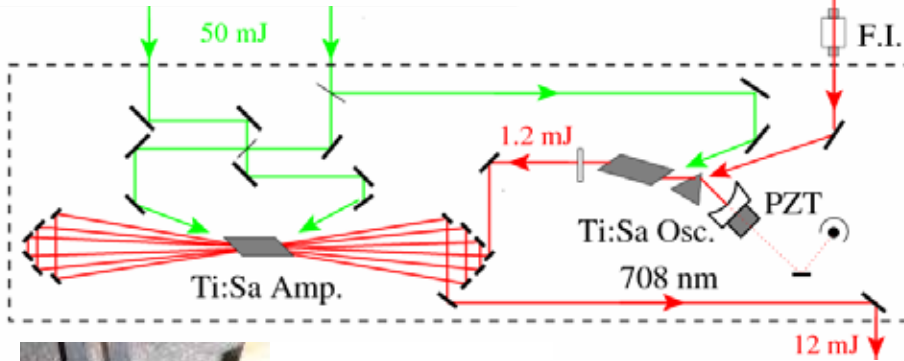
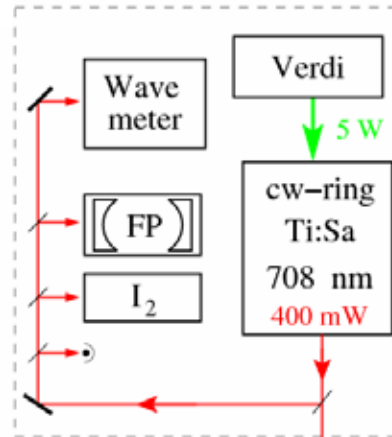
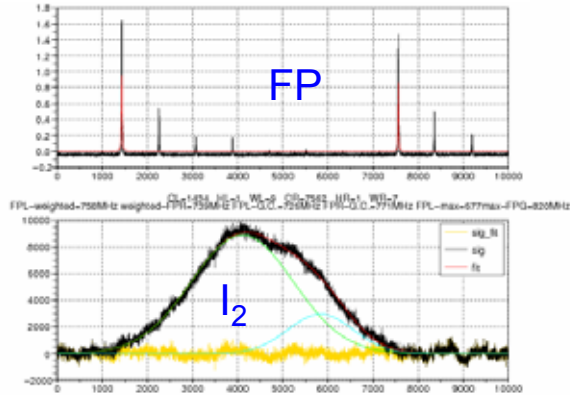
Disk amplifier



A. Antognini et al,
IEEE J. of Q. Electronics
vol 45, n°8, 2009

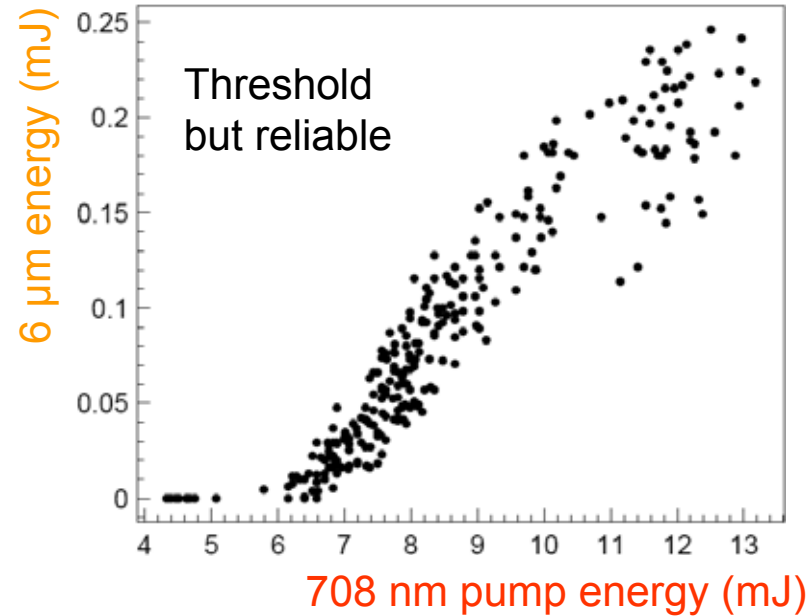
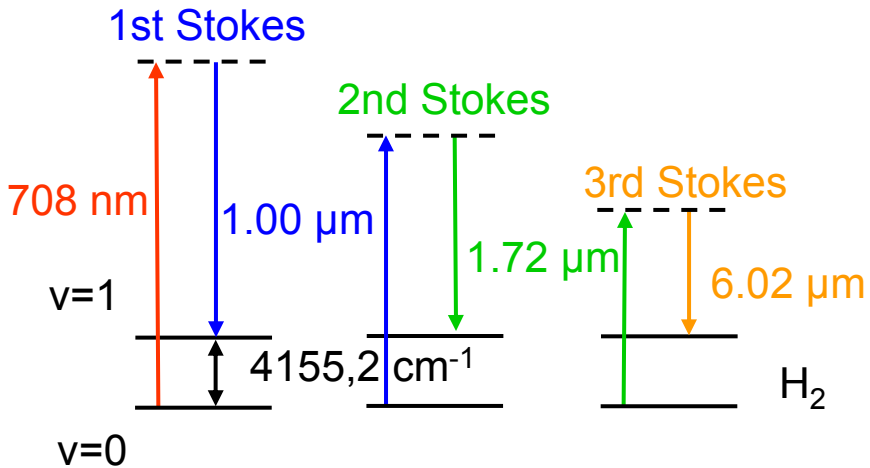
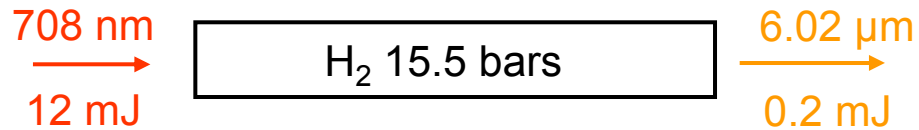
Laser chain : Ti:Sa

Calibration FP/ I₂, Cs, Rb
(700-794nm)

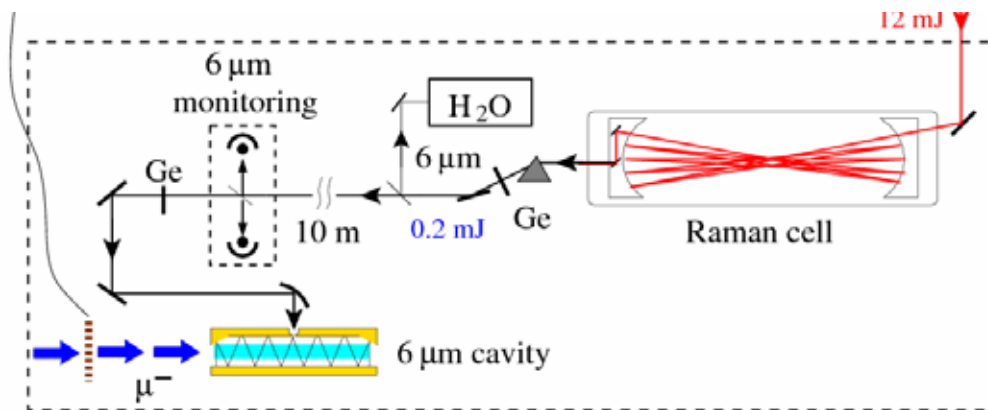
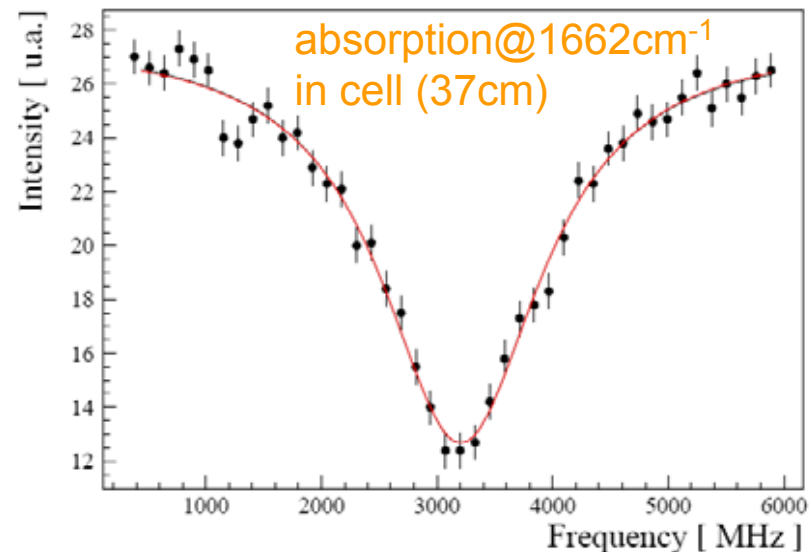


- home made Ti:Sa lasers
- cw-Ti:Sa frequency controlled with FP, atom, molecule (abs. freq. <50MHz)
- short length pulsed oscillator seeded with Cw-Ti:Sa (1.2 mJ, 6 ns, delay 50 ns, $\Delta\nu = 200\text{MHz}$)
- multipass amplifier $\times 6$ (12 mJ, 5 ns)

Laser chain : Raman cell

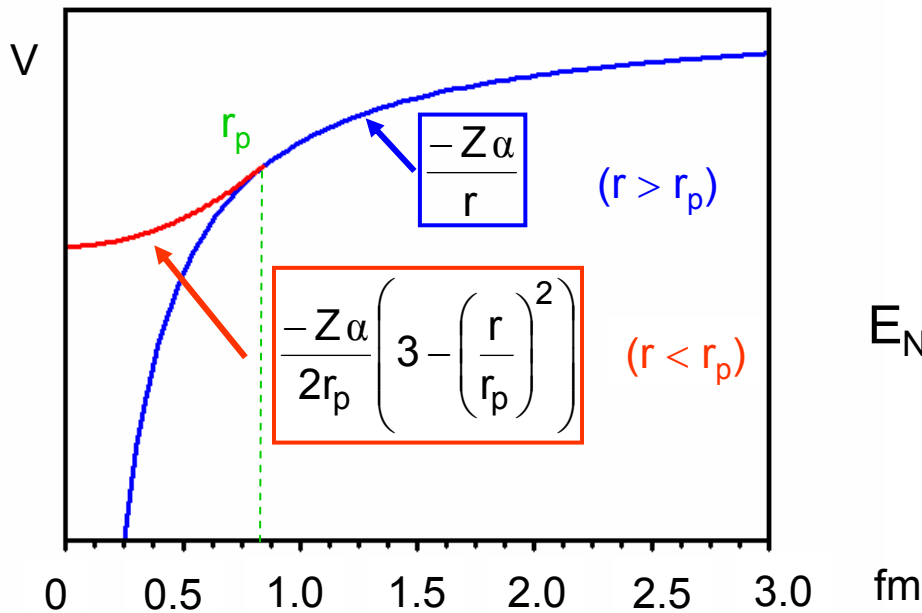


6 μm frequency calibration : H₂O lines



proton charge radius hydrogen spectroscopy

- hydrogen spectroscopy (1989, ..., 2008)
- muonic hydrogen spectroscopy (2009)



$$E_{\text{NS}} = \frac{2}{3} \left(\frac{\mu}{m_e} \right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_N}{\lambda_c} \right)^2 \delta_{10}$$

Muonic hydrogen Lamb shift

Name	Reference	formula	value	Pachucki [1]	Borie [3]
NR One loop electron VP		$\frac{8\alpha(Z\alpha)^2}{5\pi n^3} Q_{nl}^{(1)} m_r$	205.00740	205.0074	
Relativistic one loop VP	[3]				205.0282
NR two-loop electron VP		$\frac{4\alpha(Z\alpha)^2}{\pi^2 n^3} Q_{nl}^{(2)} m_r$	1.50790	1.5079	1.5081
NR three-loop electron VP	[12]	$0.08353(1) \frac{\alpha^3(Z\alpha)^2}{\pi^3} m_r$	0.00530	check	
Polarization insertion in two coulomb lines	[4,1]	$0.01244 \frac{\alpha^2(Z\alpha)^2}{9\pi^2} m_r$	0.15090	0.1509	0.1510
Polarisation insertion in two and three coulomb lines	[12]	$0.036506(4) \frac{\alpha^3(Z\alpha)^2}{\pi^3} m_r$	0.00230		
Relativistic correction	[2,4,1,3]	$\alpha(Z\alpha)^4$	0.01690	0.0169	
Wichmann-Kroll	[13,14,3]	$\frac{4\alpha(Z\alpha)^4}{\pi n^3} Q_{nl}^{WK} m_r$	-0.00103		-0.00103
Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	[4,1]		-0.00500	0.0010	-0.006
Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$		$\left[1.09426 \left(-\frac{m}{8m_r} - 1 \right) - 4 \times 2.21656 \right] \times \frac{\alpha^2(Z\alpha)^4}{\pi^2 n^3} \left(\frac{m_r}{m} \right)^5 m$	-0.00150		
Mixed electron and muon loops	[15]	$\frac{64\alpha^2(Z\alpha)^2}{45\pi^2 n^3} \left(\frac{m_e}{m} \right)^2 Q_{nl}^{(3)} m_r$	0.00007		0.00007
Hadronic polarization $\alpha(Z\alpha)^4 m$	[16]	$-0.638(22) \frac{4\alpha(Z\alpha)^4}{15\pi^2 n^3} \left(\frac{m_r}{m} \right)^5 m \delta_{10}$	0.01080	0.0004	0.0113
Hadronic polarization $\alpha(Z\alpha)^5 m$			0.00005		0.0110
Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m$			-0.00002		
Recoil contribution	[17]	$\frac{(Z\alpha)^4 m^3}{2n^3 M^2} \left(\frac{1}{j+\frac{1}{2}} - \frac{1}{l+\frac{1}{2}} \right) (1 - \delta_{10})$	0.05750	0.0575	0.0575
Radiative corrections of order $\alpha^n(Z\alpha)^k m$	[11]	Tables 3.2, 3.3, 3.7, 3.8, 3.9	-0.66770	-0.6677	-0.66788
Recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m$	[11]	Table 4.1	-0.04470	-0.045	-0.04497
Radiative Recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m$	[11]	Table 5.1	-0.00960	-0.0099	-0.0096
Nuclear structure correction of order $(Z\alpha)^5$	[1]	$-\frac{0.095(18)}{n^3} \delta_{10} \text{ meV}$	0.01200		0.015
Light by light electron loop contribution			0.00135	0.00135	
Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m$		$-\frac{0.00152}{n^3} \delta_{10} \text{ meV}$	0.00019		
Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m$		$\frac{0.000092}{n^3} \delta_{10} \text{ meV}$	-0.00001		
Sum			206.04310	0.00275	206.0353
terms dependent on the nuclear radius from Ref. [11] (contributions in meV)					
Leading nuclear size contribution		$\frac{2(Z\alpha)^4 m_r^3}{5n^3} \langle r^2 \rangle \delta_{10}$	-5.1975	$\langle r_p^2 \rangle$	
Nuclear size correction of order $(Z\alpha)^5$	[4,1]	$-\frac{(Z\alpha)^5 m_r^4}{3n^3} \langle r^3 \rangle \delta_{10}$	0.0347	$\langle r_p^3 \rangle$	0.0347 $\langle r_p^3 \rangle$
Radiative corrections to nuclear finite size effect	[1]	$\alpha(Z\alpha)^4 m_r^3 \langle r_p^2 \rangle$	-0.0275	$\langle r_p^2 \rangle$	-0.0273 $\langle r_p^2 \rangle$
Nuclear size correction of order $(Z\alpha)^6 \langle r_p^2 \rangle$	[1]	$-\langle \ln \frac{Z\alpha}{2} \rangle \frac{2(Z\alpha)^6}{3n^3} m_r^3 \langle r_p^2 \rangle$		$\langle r_p^2 \rangle$	
Nuclear size correction of order $(Z\alpha)^6 \langle r_p^4 \rangle$	[1]	$\frac{2}{3} \frac{2(Z\alpha)^6}{5n^3} m_r^5 \langle r_p^4 \rangle$		$\langle r_p^4 \rangle$	
Total $\langle r^2 \rangle$ contribution			-5.225		

Form factor (dipole, Gaussian, ...)

d:4, G:1

Table 7. All known contributions from Eides et al.[11])

Hydrogen spectroscopy

Transition energy : $(E_n - E_p) = (E_n^{Dirac} - E_p^{Dirac}) + (E_n^{Recoil} - E_p^{recoil}) + (L_n - L_p)$

in which : E_{Dirac} and E_{Recoil} have an exact formulation prop. to R_∞
 $L_n = F(\text{QED}, R_p)$ Lamb shift

✓ Q.E.D. = series expansion of α , $Z\alpha$, $\ln(Z\alpha)$ which depends on n as $1/n^3$

$$\Delta E = \left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^4}{n^3} m_e c^2 \left[B_{40} + (Z\alpha) B_{50} + (Z\alpha)^2 (L^3 B_{63} + L^2 B_{62} + L B_{61} + B_{60}) + \dots \right]$$

$$\approx \dots \left[0,54 - 0,16 + 5 \times 10^{-5} (-280 - 50 + 500 - 100) + \dots \right]$$

avec $L = \ln[(Z\alpha)^{-2}]$

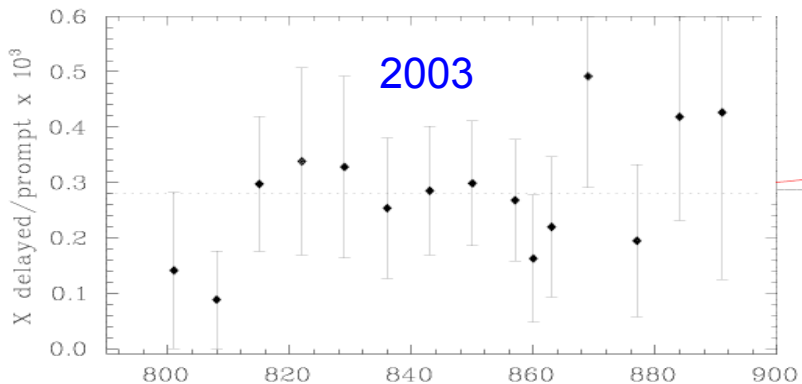
✓ Distribution charge radius contributions vary as $1/n^3$

$$E_{NS} = \frac{2}{3} \left(\frac{\mu}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_p}{\lambda_c}\right)^2 \delta_{l0}$$

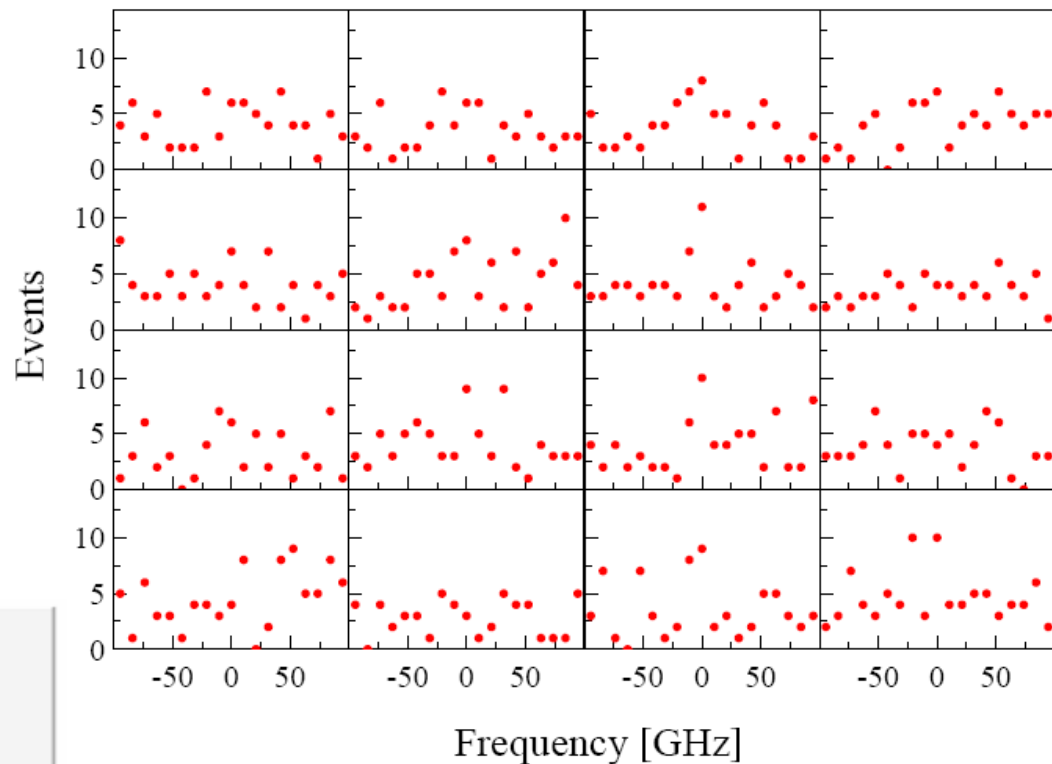
- Q.E.D. calculations are right ! : 2 experimental data $\rightarrow R_\infty, R_p$
- are Q.E.D. calculations right ? : 3 experimental data $\rightarrow R_\infty, R_p, \text{Q.E.D.}$

Resonance searches (2002, 2003, 2007)

2002 : Run with all the components of the experiment at the same time (search for resonance 10 hours).

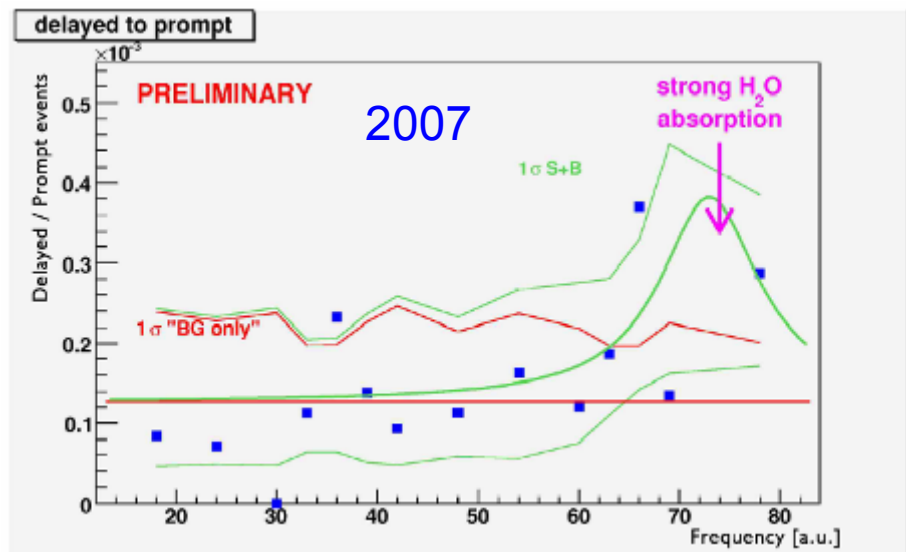


Simulation 2003 :

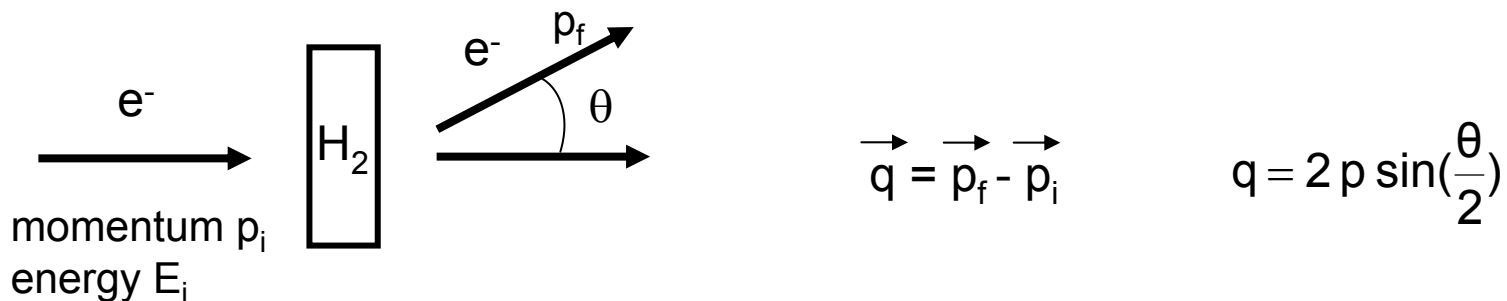


sig. 0.5 c/hour
noise 0.5 c/hour

from A. Antognini Phd thesis



electron-proton scattering



$$\frac{d\sigma}{d\Omega}(E_i, \theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i, \theta) \right]}_{\text{Rutherford}} G(q^2)$$

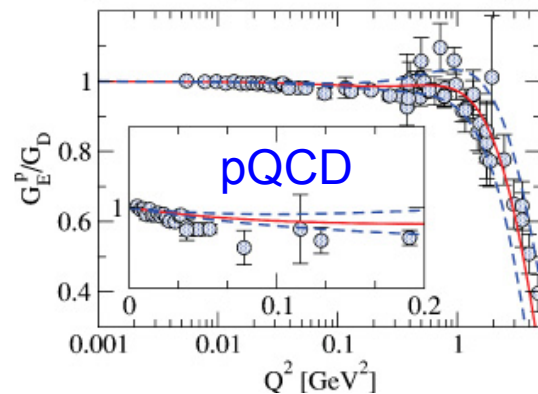
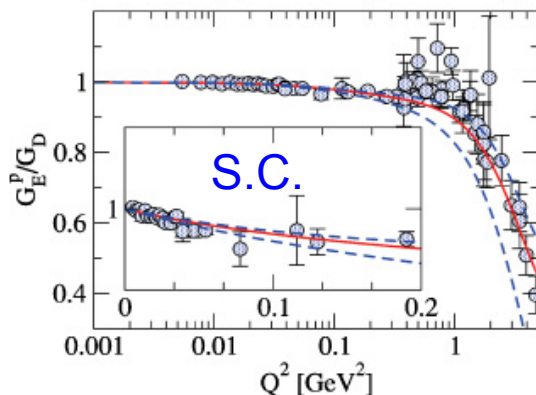
$$G(q^2) = \int d^3r e^{iqr} \frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6} q^2$$

Sick 2003 :

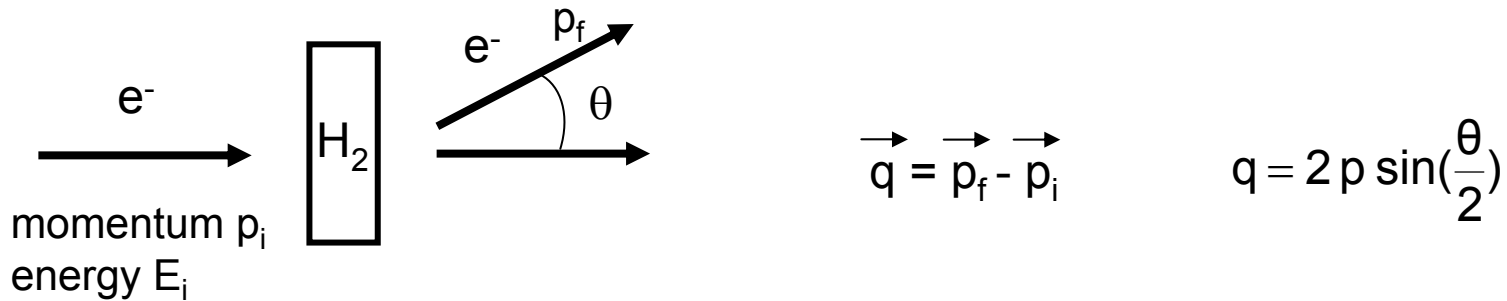
$$G_e(q) = \frac{1}{1 + \frac{q^2 b_1}{1 + \frac{q^2 b_2}{1 + \dots}}}$$

Belushkin et al 2007 :

$$G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{m_D^2}\right)^2}$$

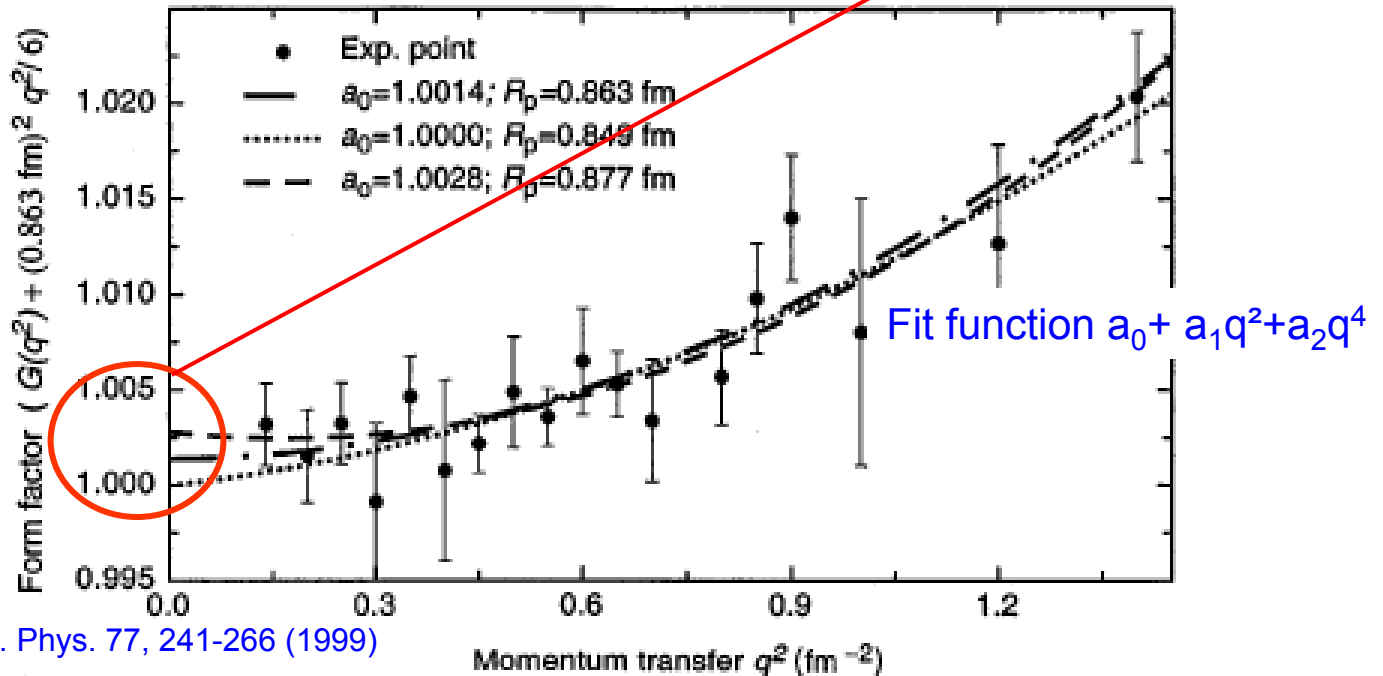


electron-proton scattering



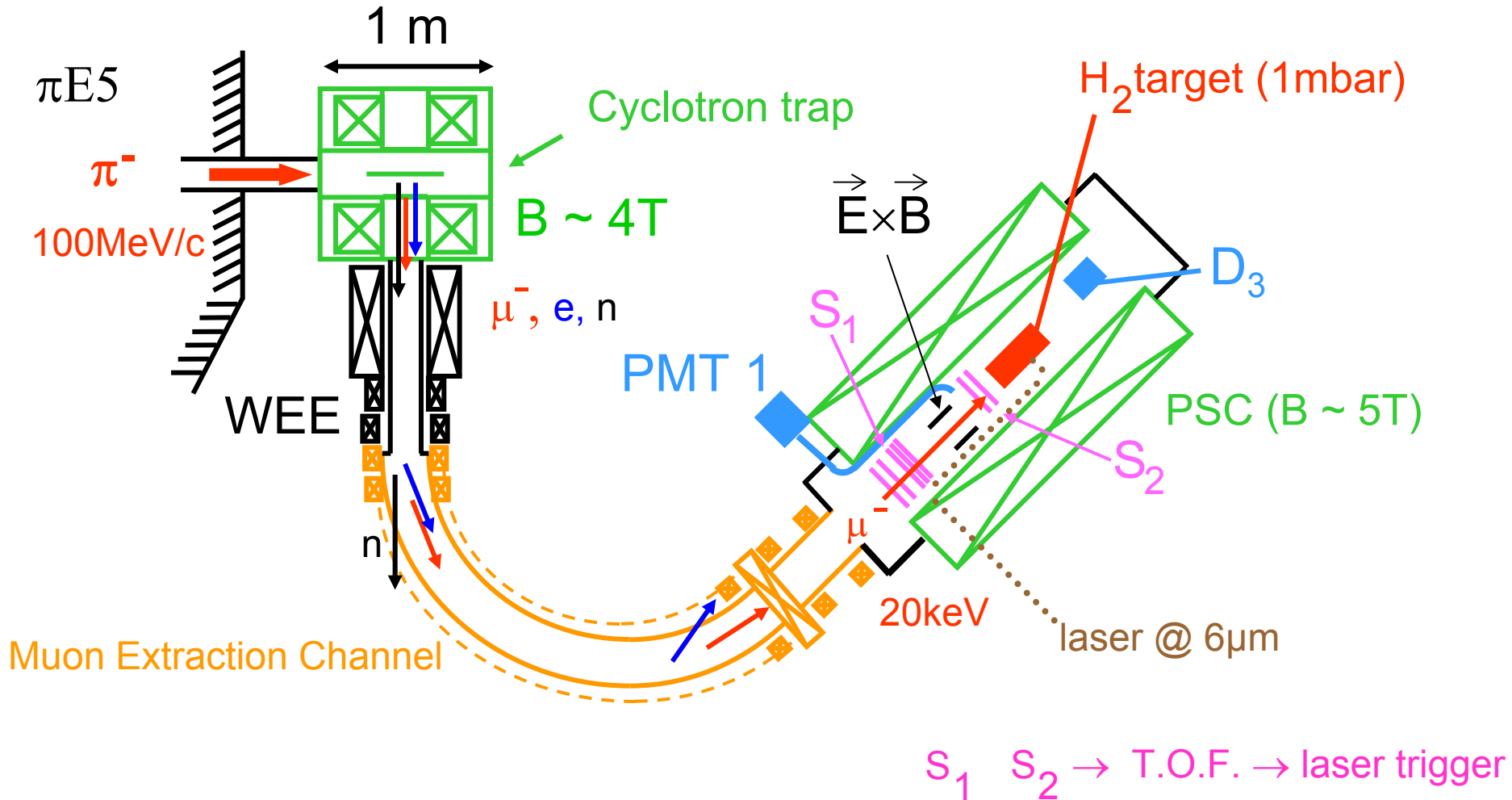
$$\frac{d\sigma}{d\Omega}(E_i, \theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i, \theta) \right]}_{\text{Rutherford}} G(q^2)$$

$$G(q^2) = \int d^3r e^{iqr} \frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6} q^2$$



muonic hydrogen source

p (500MeV, 2mA) \rightarrow carbon target $\rightarrow \pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ (300 μ /s)

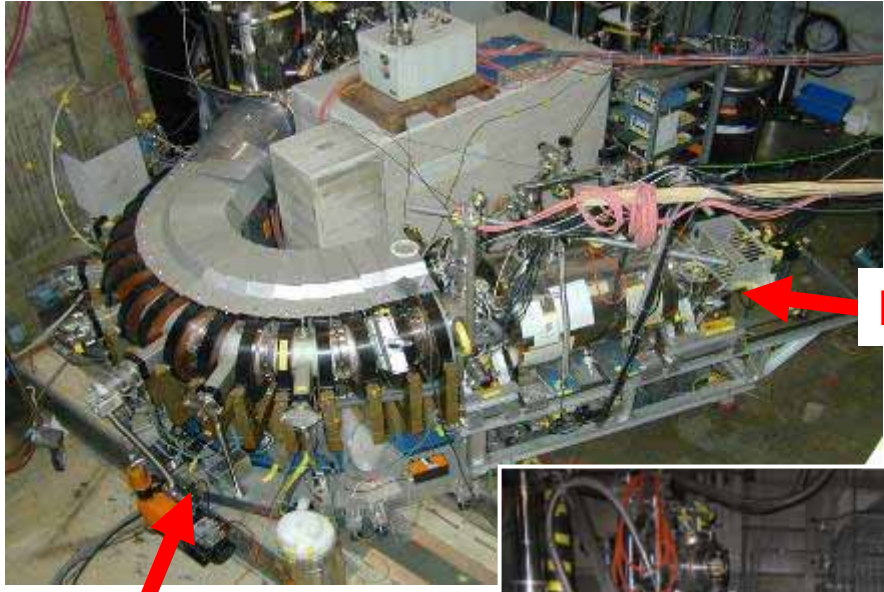
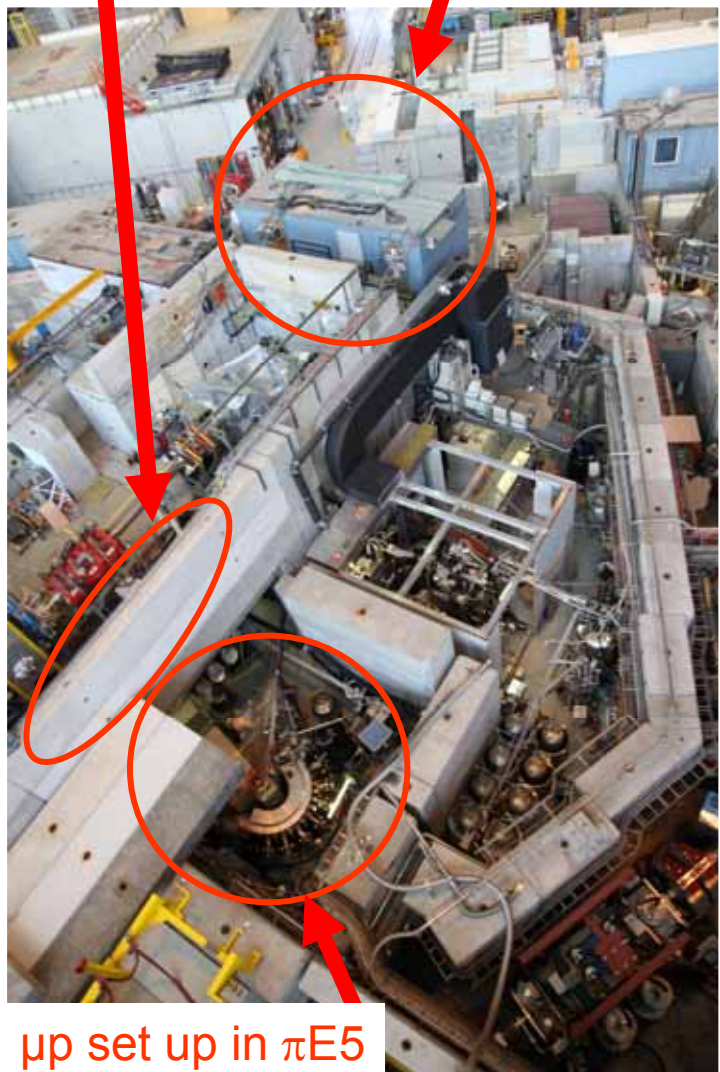


muon beam apparatus



laser hut
below
concrete blocks

counting room

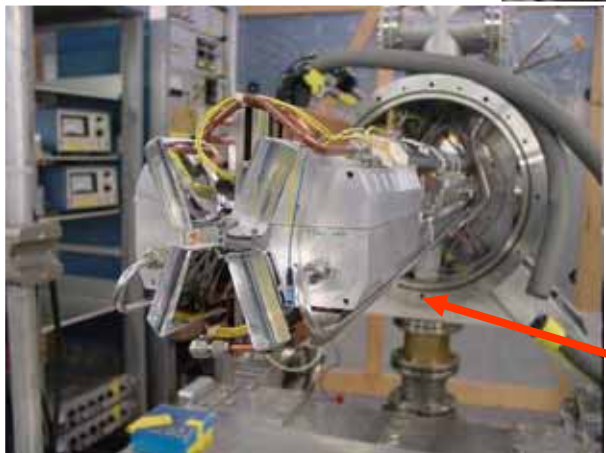


PSC solenoid,

Muon extraction
channel

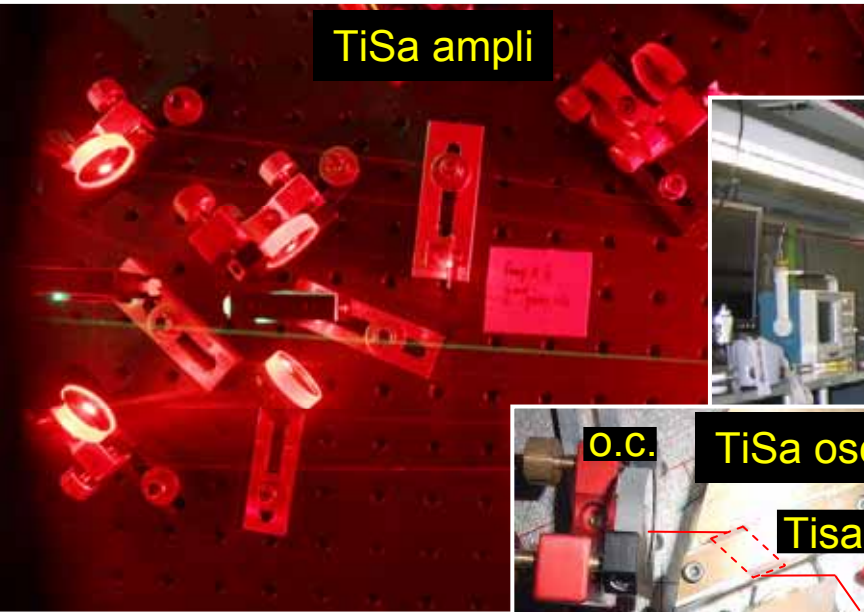


H2 target, laser cavity,
detectors

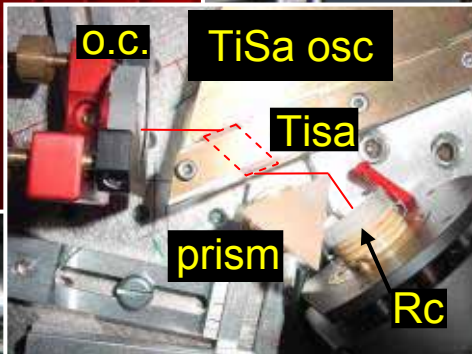
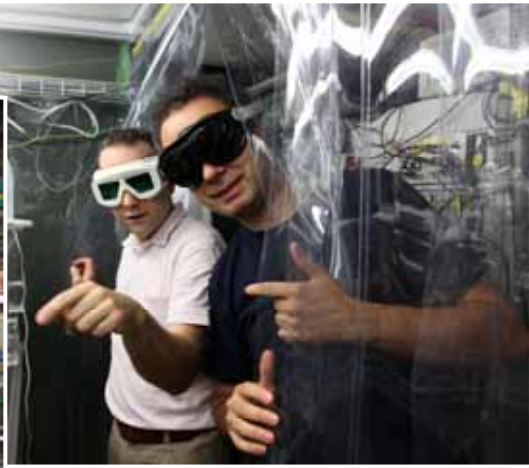
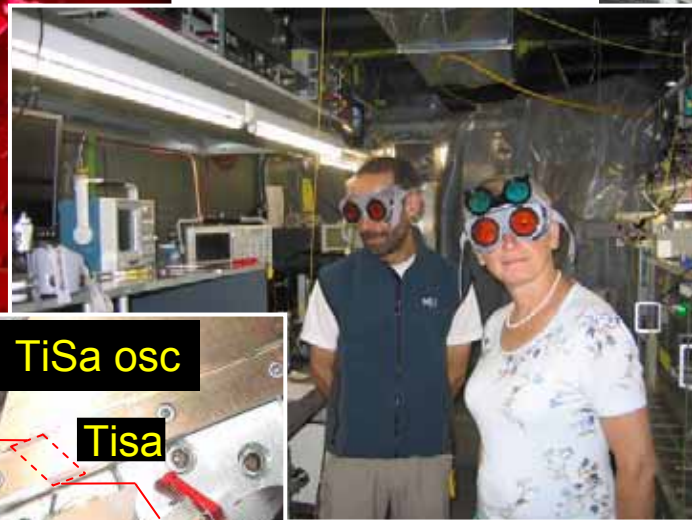


μp set up in $\pi E5$

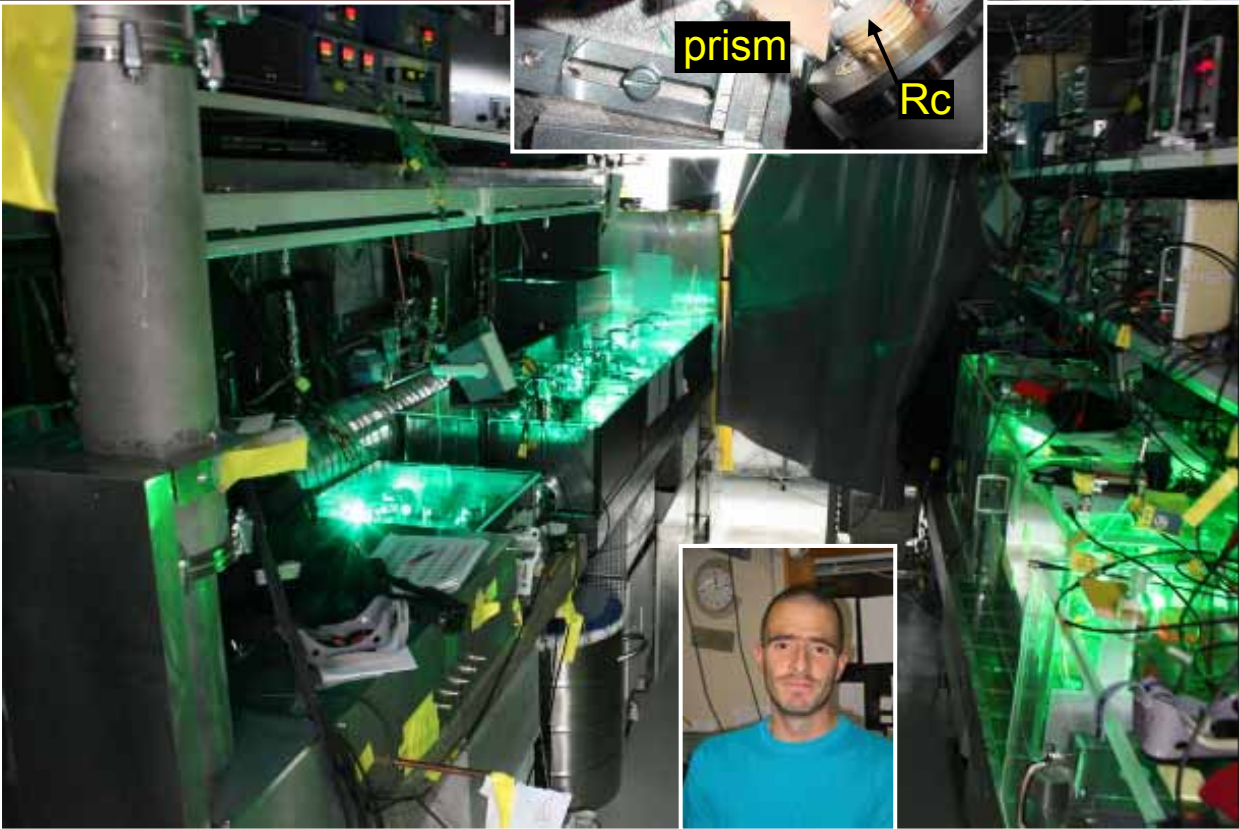
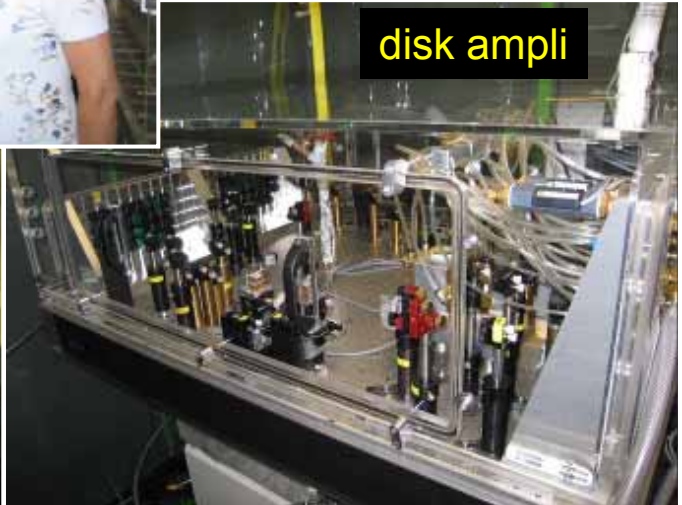
TiSa ampli



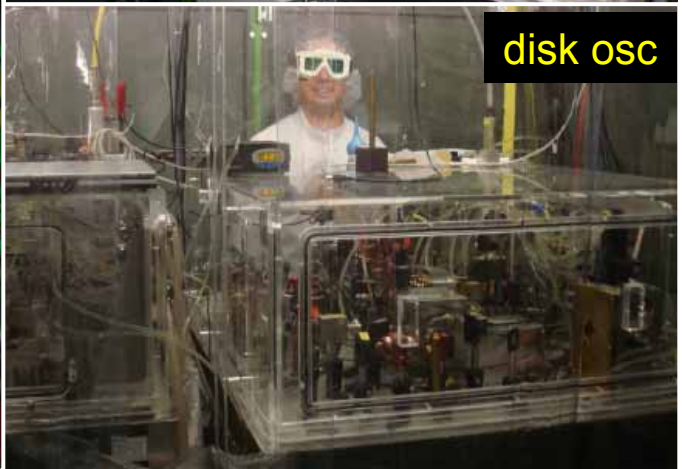
Laser hütte



disk ampli

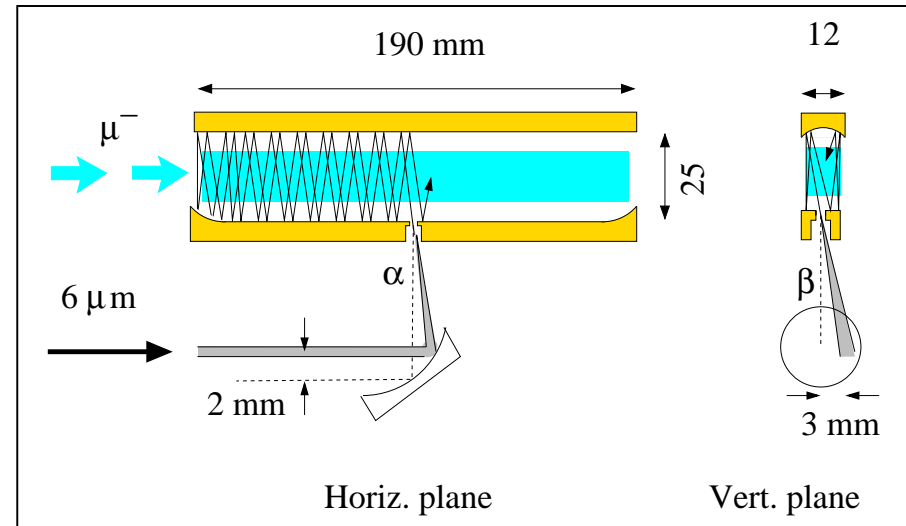
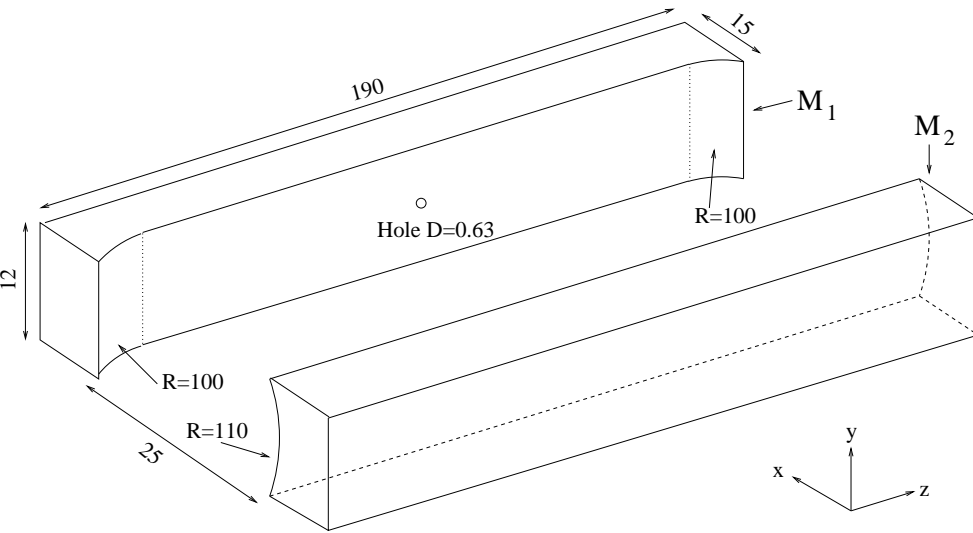


disk osc

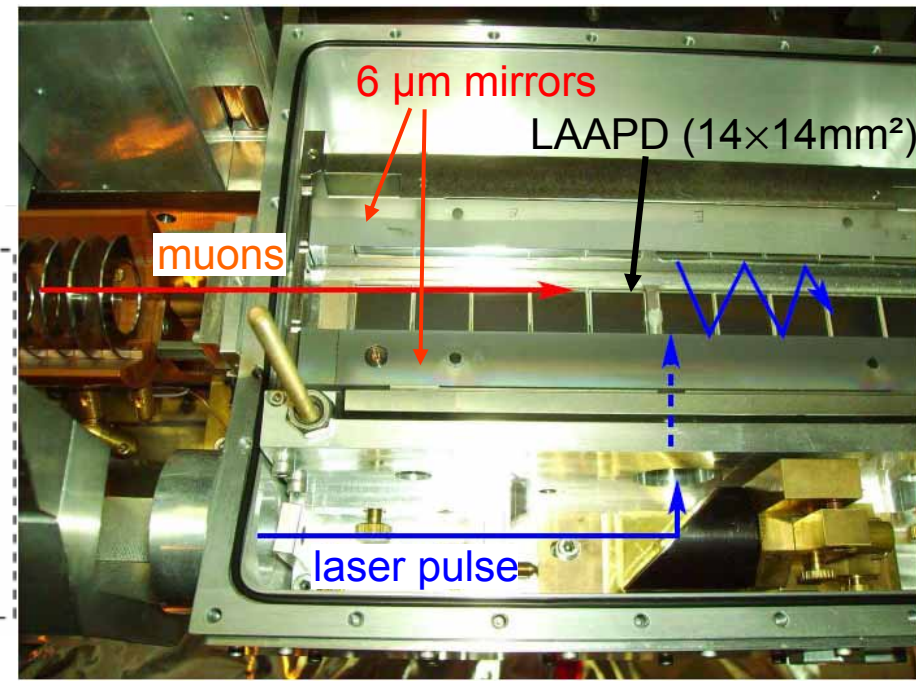
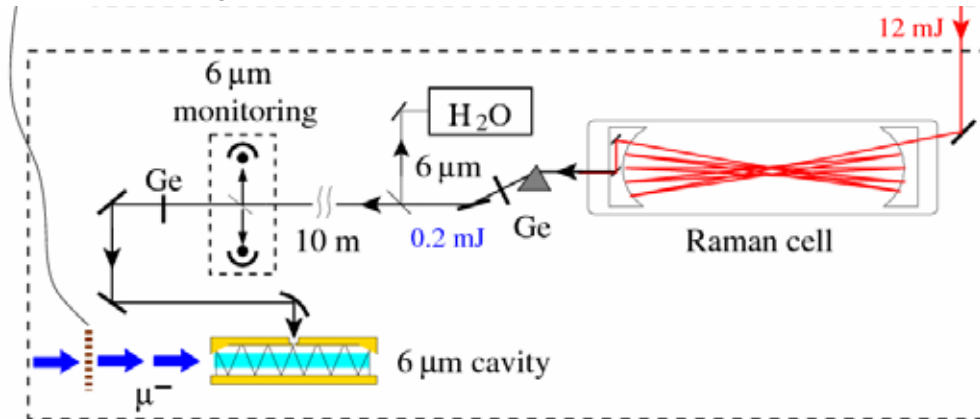


Laser chain : multipass cavity

→ illuminate at 6 μm all the muon stopping volume ($5 \times 15 \times 190 \text{ mm}^3$)



- coupling through a 0.63mm diameter hole
- $R=99.90\%$ at 6 μm
- 1000 reflections
- 0.15mJ injected → 2S-2P saturated

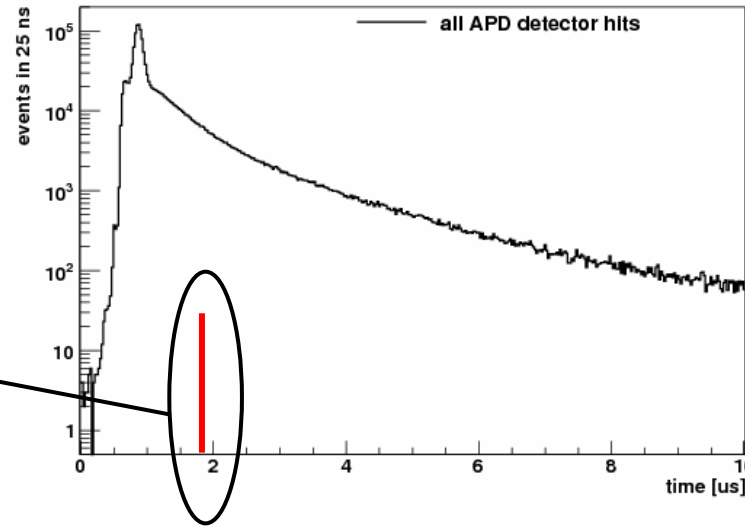


X-rays analysis → event gate sorting → noise rejection

Example : FP 900 - 11 hrs meas.

1.56 million detector events

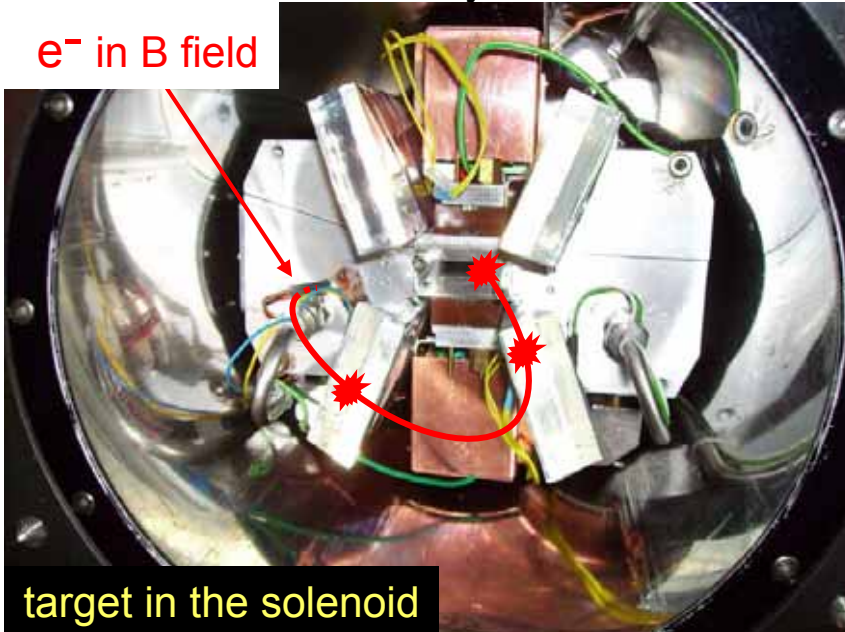
expected 2-3 laser induced events/hour !



time signature in LAAPD

- photon $< 10\text{keV}$ → 1 shot in the LAAPD
- e^- in $B = 5\text{T}$ → many counts in detectors

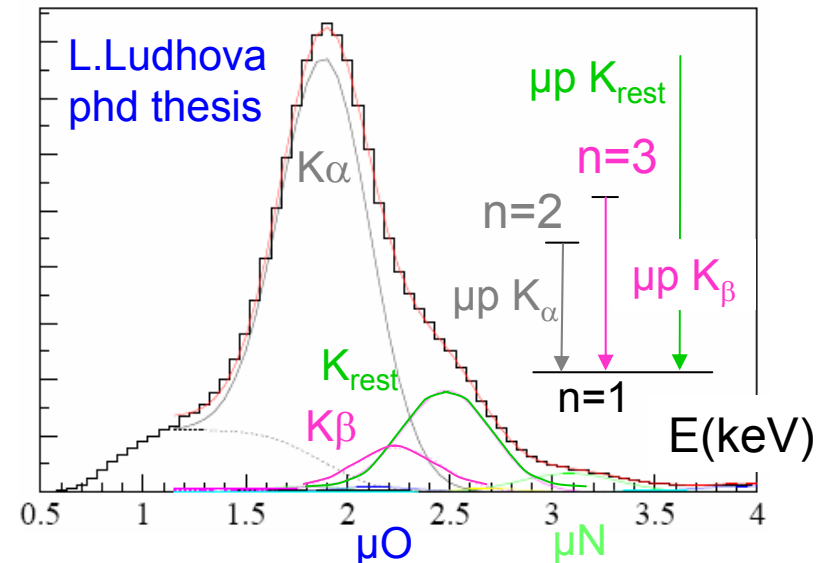
e^- in B field



target in the solenoid

energy signature in LAAPD

- $E > 8\text{keV}$ ⇔ electron
- $1\text{keV} < E < 8\text{keV}$ ⇔ X ray
- $E < 1\text{keV}$ ⇔ neutron

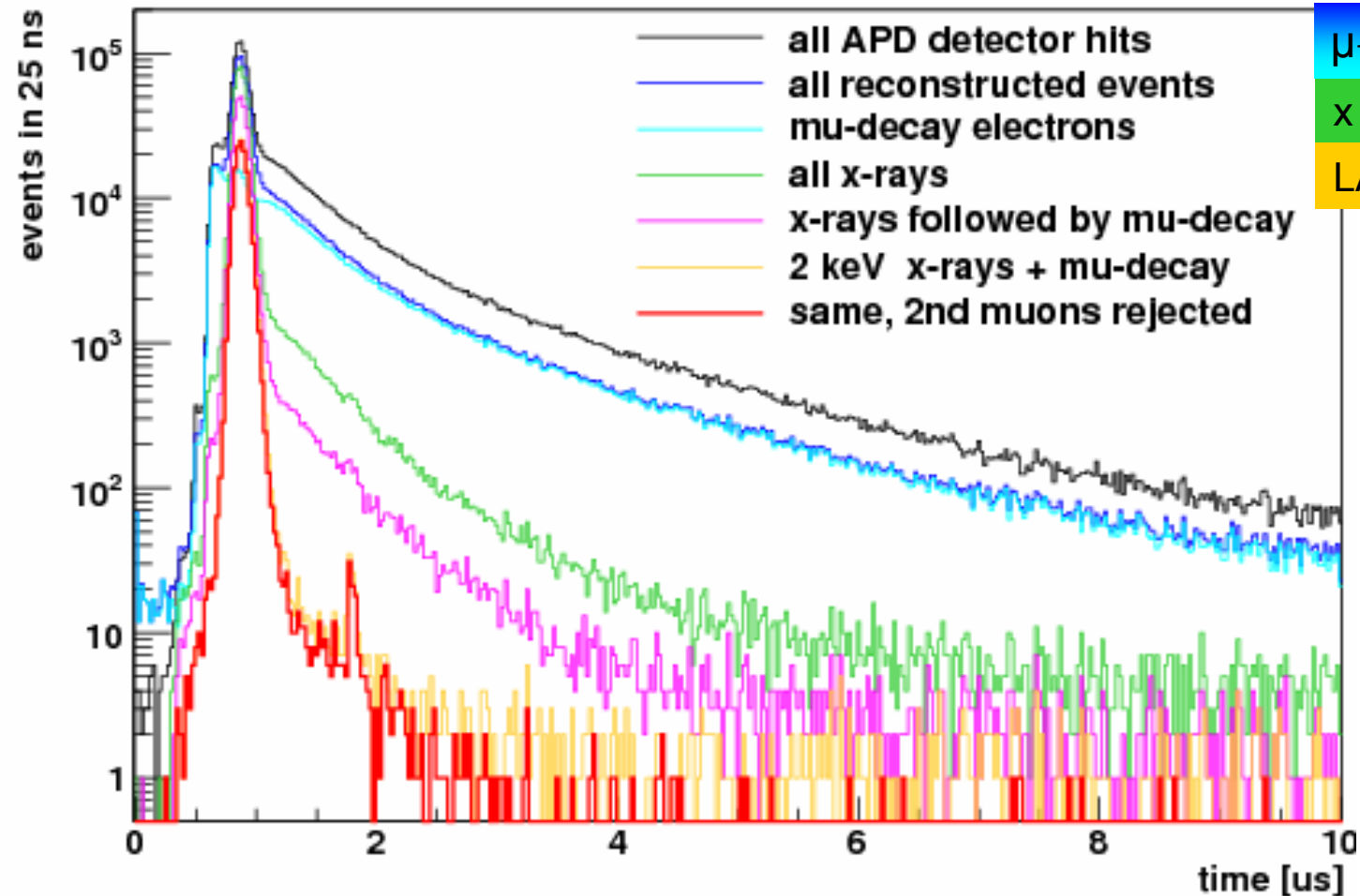


L.Ludhova
phd thesis

X-rays analysis → noise rejection

Example : FP 900 - 11 hrs meas.

- 400 μ^- /s
- 240 laser shot/s
- 860 000 laser shot/hour
- 1.56 million detector clicks
- 19600 clicks in the laser region
- expected 2-3 laser induced events/hour !



$\mu \rightarrow e \nu_{\mu} \bar{\nu}_e$

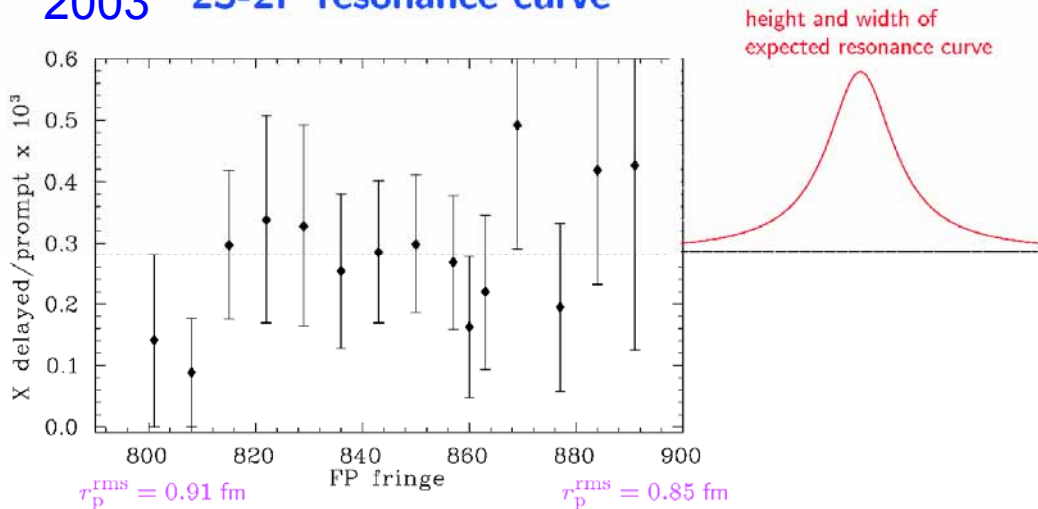
x rays multiplicity 1

LAAPD energy resolution

Resonance searches (2002, 2003, 2007)

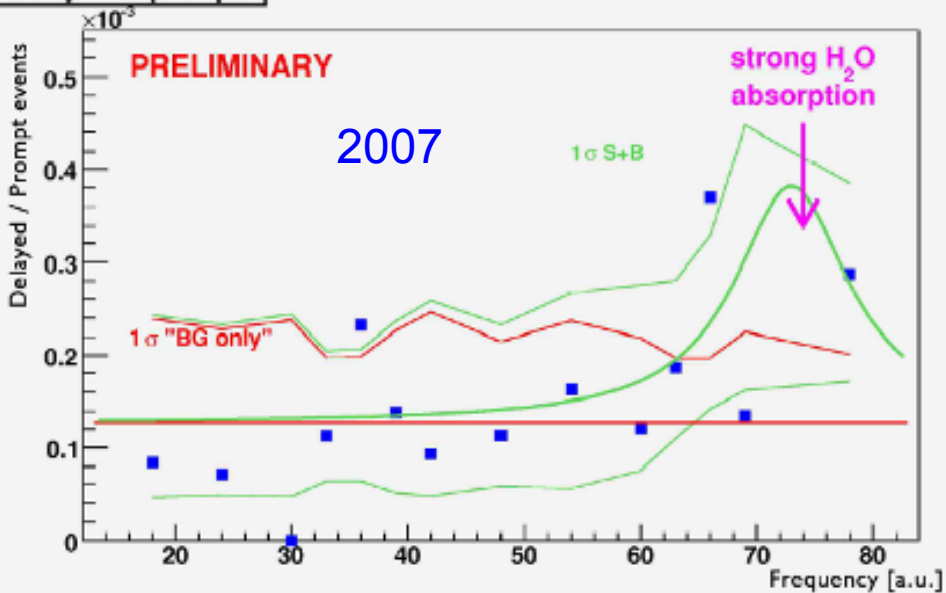
2002 : Run with all the parts at the same time (search for resonance 10 hrs).

2003 2S-2P resonance curve



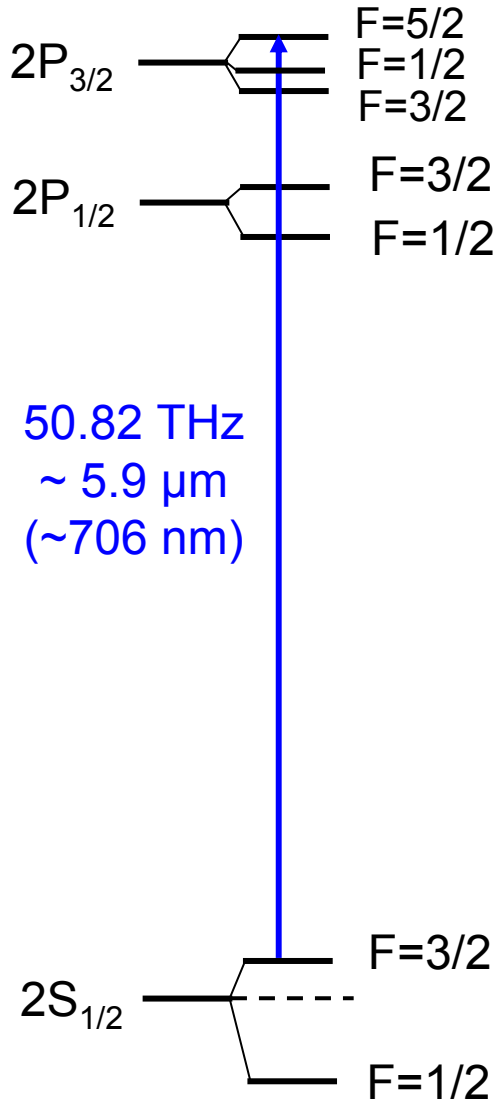
2 “new” XeCl lasers

delayed to prompt

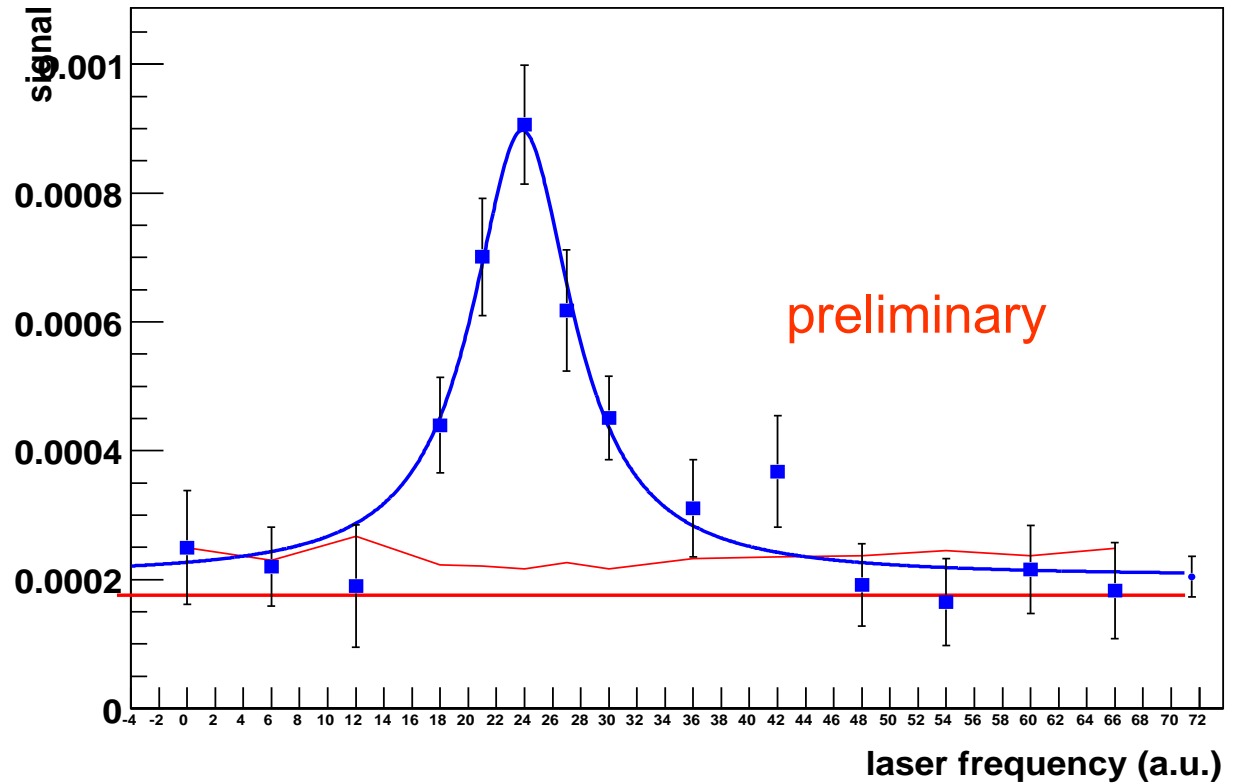


new disk laser

muonic deuterium : $2S_{1/2}(F=3/2) - 2P_{3/2}(F=5/2)$

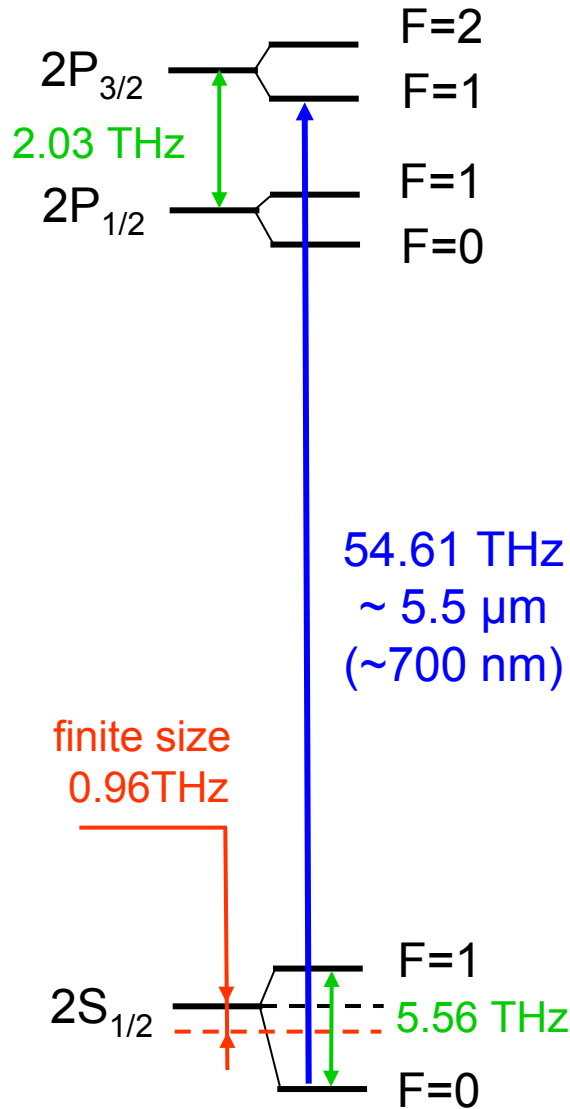


- on line signal, **preliminary**
- frequency position again off

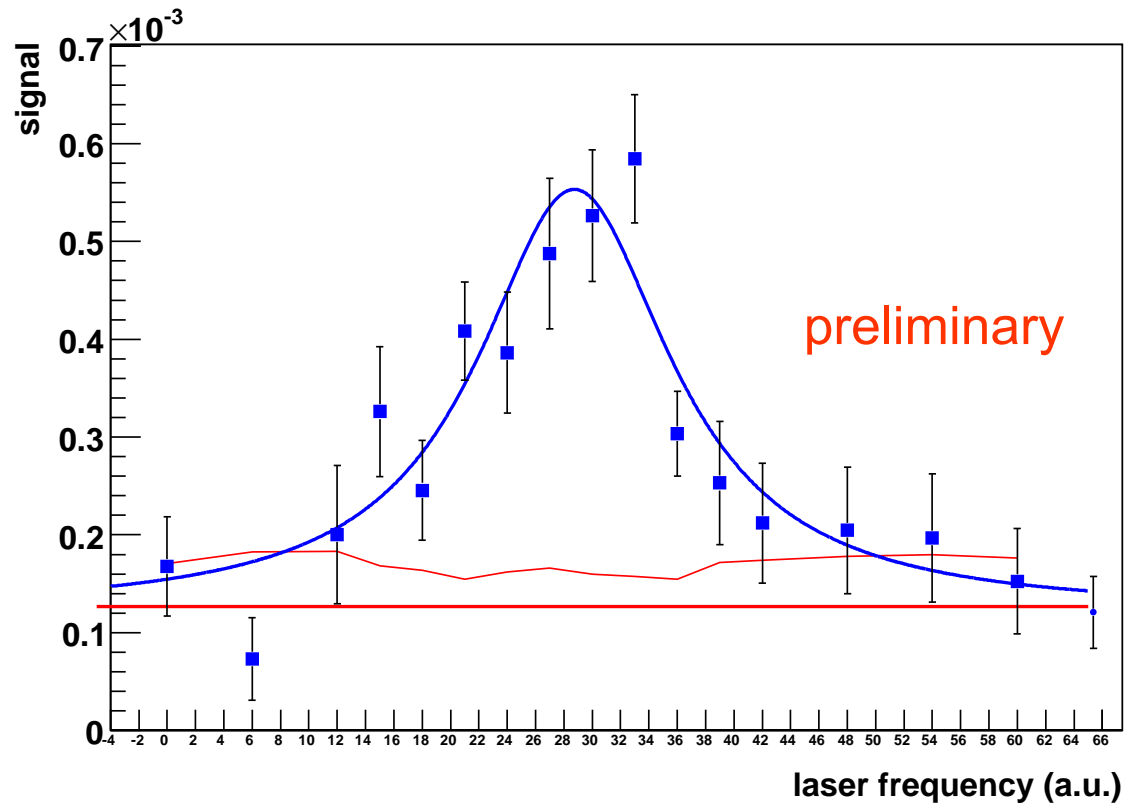


Not at the position estimated with new r_p and isotopic shift \rightarrow deuteron polarizability

muonic hydrogen: $2S_{1/2}(F=0)$ - $2P_{3/2}(F=1)$

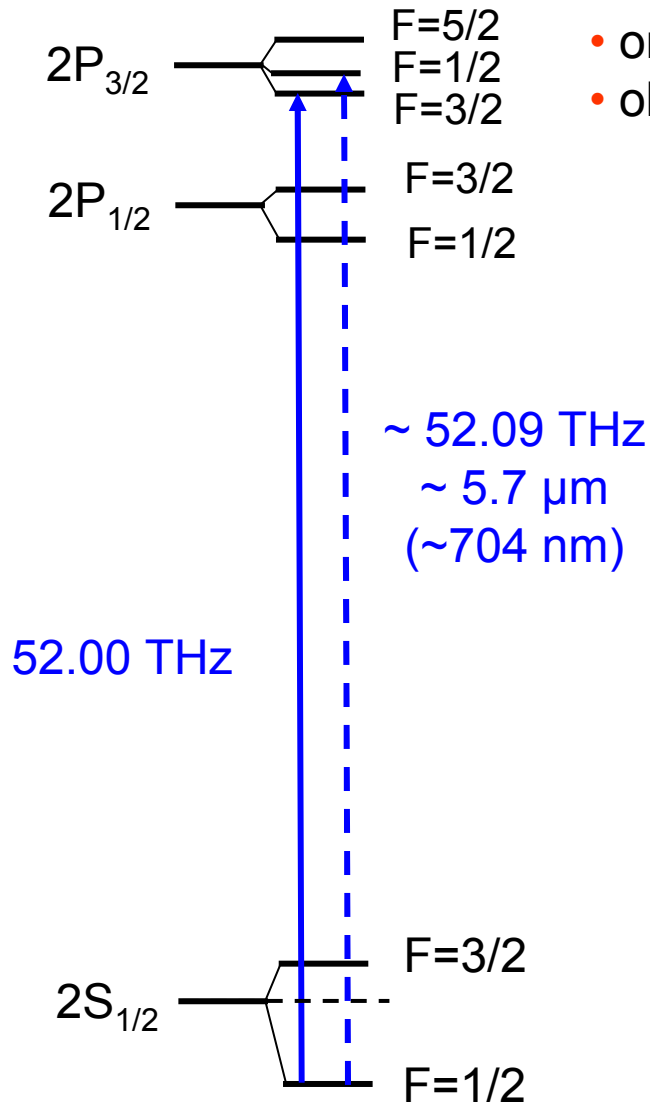


- measured position fits with our proton radius (**preliminary**)
- laser worked even better at $5.5 \mu\text{m}$

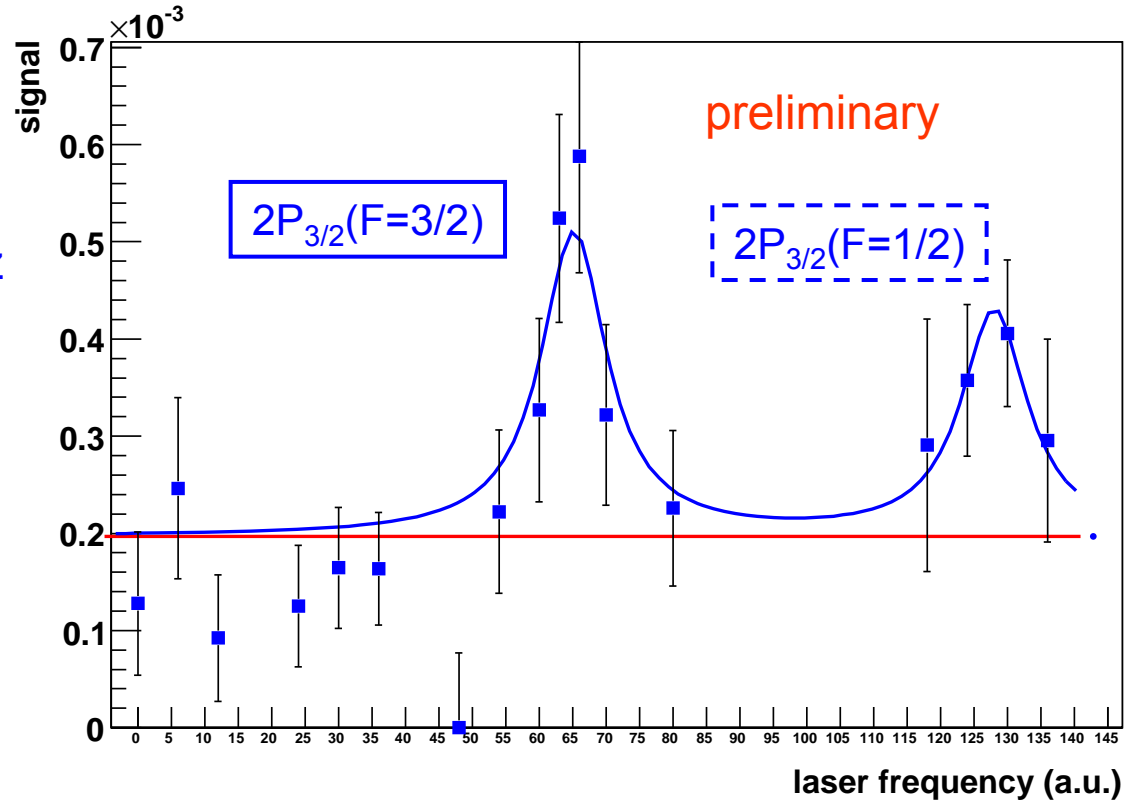


\sim at the position deduced with new $r_p \rightarrow$ hfs : Zeemach radius (few %)

muonic deuterium : $2S_{1/2}(F=1/2)$ - $2P_{3/2}(F=3/2)$ and $2S_{1/2}(F=1/2)$ - $2P_{3/2}(F=1/2)$



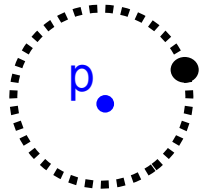
- on line signal, **preliminary**
- observation of 2 more lines in the last 2 days of beam line



“observation” → check calculation in μd

Muonic hydrogen spectroscopy

Exotic atom



$$\frac{m_{\mu}}{m_e} = 207$$

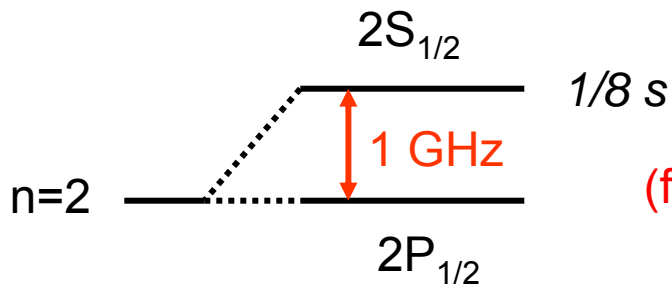
radius : $\sim a_0/200$

$$E_{NS} = \frac{4}{3} \left(\frac{m_r}{m_e} \right)^3 \frac{R_{\infty} hc}{n^3} \left(\frac{r_p}{a_0} \right)^2 \delta_{10}$$

Lamb shift = self-energy + vacuum polarization + proton radius

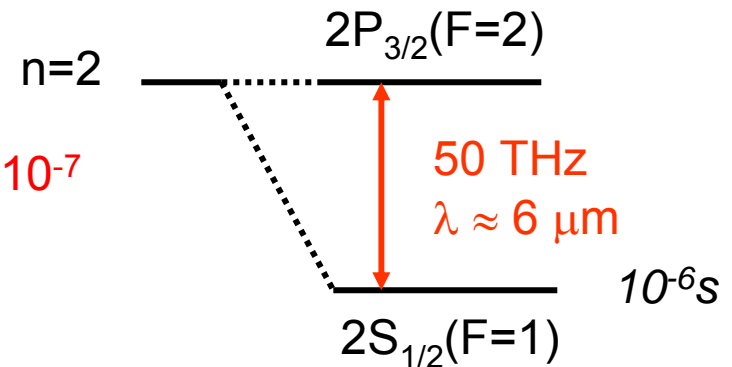
2S-2P	self-energy	vacuum pol.	r_p	total
e-p	1084.1 MHz	-26.9 MHz	0.146 MHz	1057.8 MHz
μ -p	0.17 THz	-50.94 THz	0.96 THz	-49,81 THz

electronic hydrogen : e-p



$$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \approx 10^{-7}$$

muonic hydrogen : μ -p

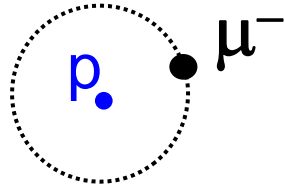


$$\Delta E_{2S-2P} = 50.7724(12) - 1.2637 r_p^2 - 0.0084 r_p^3 = 49.8083 (149) \text{ THz}$$

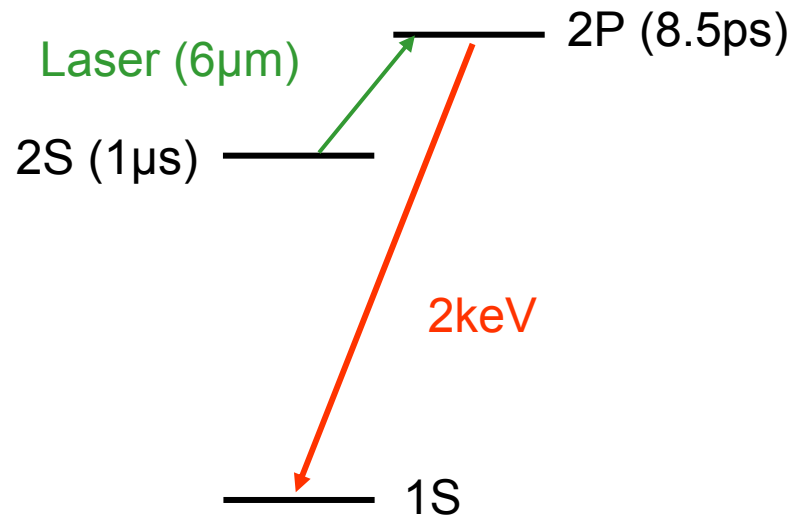
with $r_p = 0.8760(68) \text{ fm}$

muonic hydrogen 2S Lamb shift determination of the “proton radius”

Exotic atom



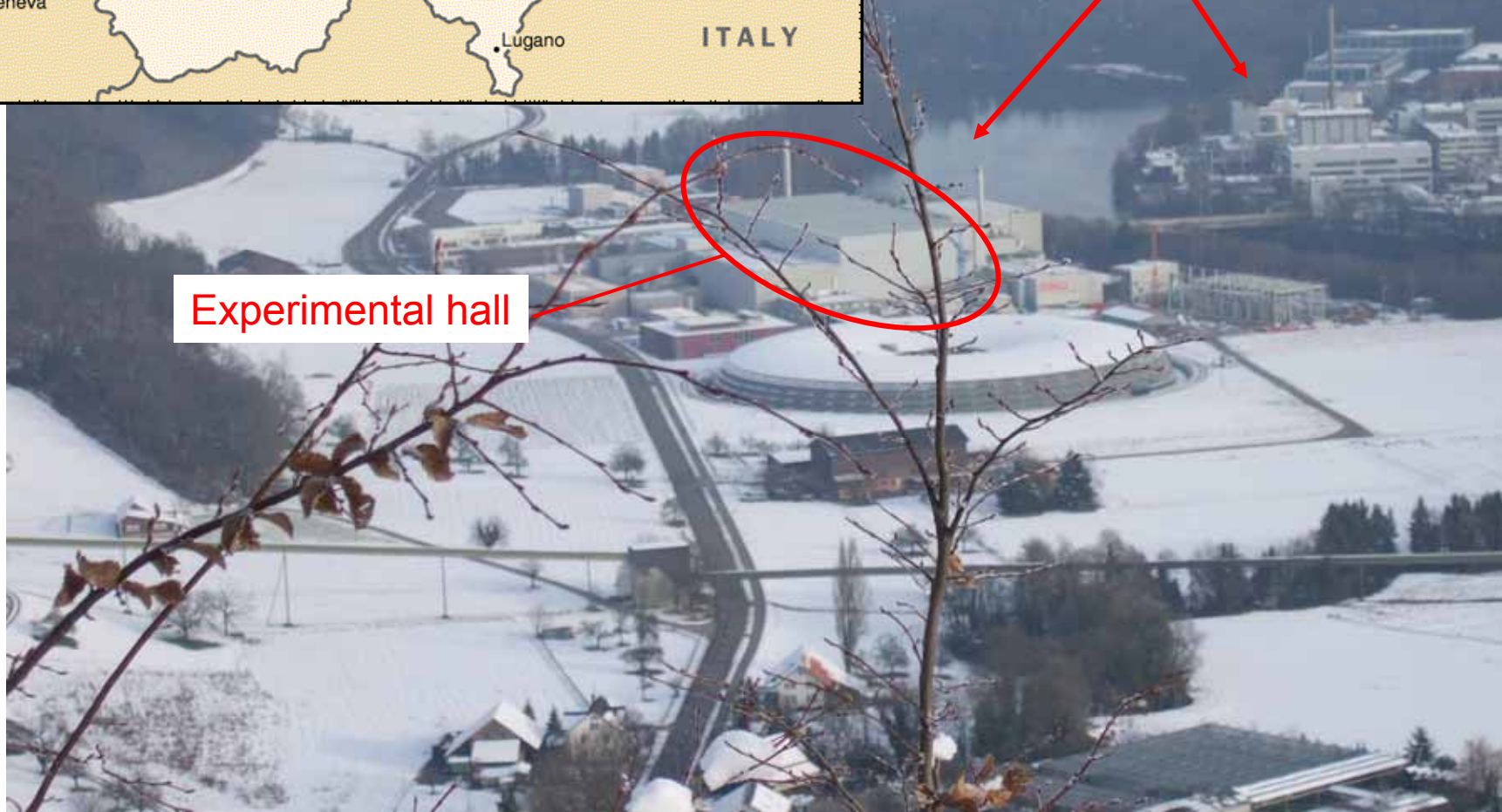
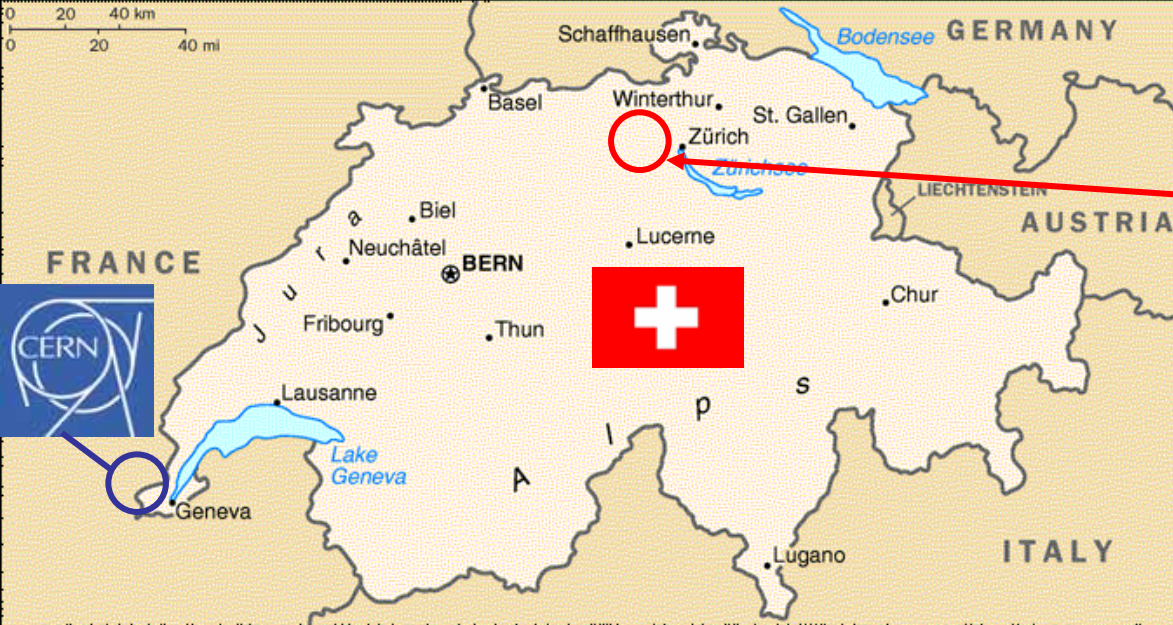
Experiment



Challenges

- production of muonic hydrogen in 2S
- powerful triggerable 6 μ m laser
- small signal analysis

Aim : better determination of proton radius r_p



Experimental hall

Outline

1. Introduction

- proton properties
- experimental proton size determinations
- muonic hydrogen

2. Experiment

- muonic hydrogen source
- laser system
- analysis

3. Results

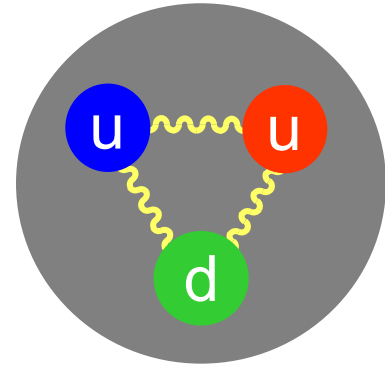
- signal
- proton radius from muonic hydrogen spectroscopy

4. Future

Proton

Structure

2 quarks **up** ($2/3 e$) + 1 quark **down** ($-1/3 e$) + strong interaction (**gluons**)



Proton charge radius : r_p

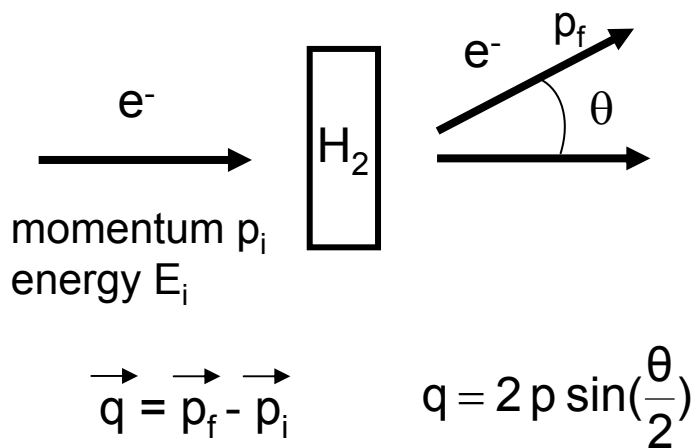
$$r_p = \sqrt{\langle r^2 \rangle} \quad \langle r^2 \rangle = \frac{\int r^2 \rho(r) d^3r}{\int \rho(r) d^3r}$$

$\rho(r)$: proton charge distribution

QCD Calculations

- proton mass (< 4%)
- r_p (< 5%)

Electron-proton scattering

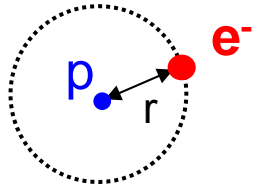
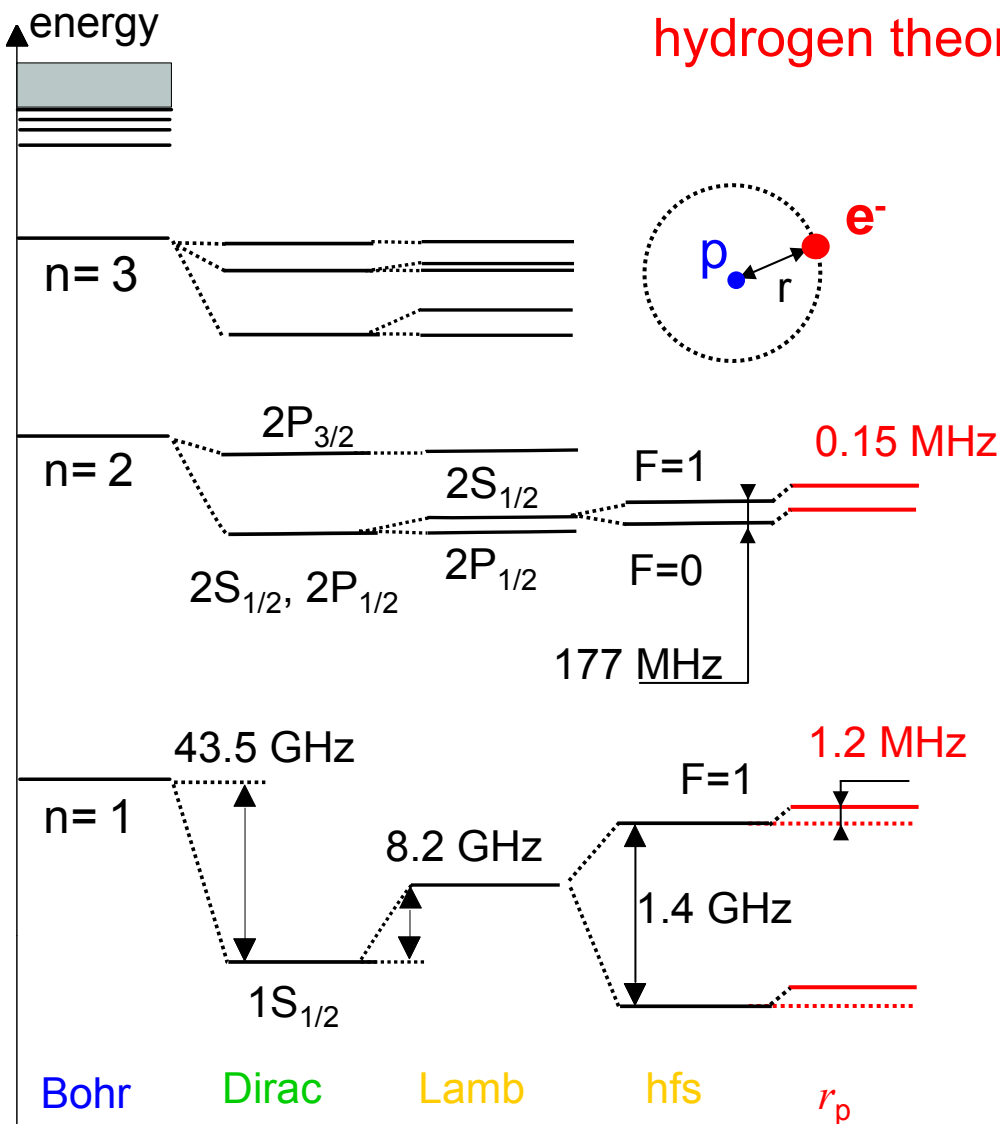


$$\frac{d\sigma}{d\Omega}(E_i, \theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i, \theta) \right]}_{\text{Rutherford}} G(q^2)$$

$$G(q^2) = \int d^3r e^{iqr} \frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6} q^2$$

Limiting factor

hydrogen theory



$$E(n,l,j) = \underbrace{\text{Dirac} + \text{recoil}}_{\text{exact}} + \underbrace{L(n,j)}_{\text{not well known}}$$

$$hcR_\infty f(\alpha, m_e/m_p, n,l,j)$$

exact

not well known

- QED corrections ($1/n^3$)
- relativistic recoil
- charge radius of the proton ($1/n^3$)

$$E_n = -\frac{R_\infty hc}{n^2}$$

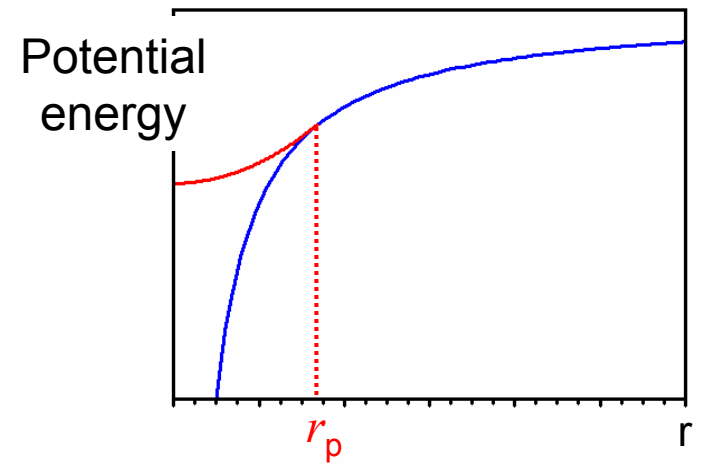
e^- spin
relativity

QED

proton
spin

proton
size

$$E(1S_{1/2}) = -3\,288\,086\,857\,127.6\ (3.1)\ \text{kHz.}$$



hydrogen spectroscopy experiments

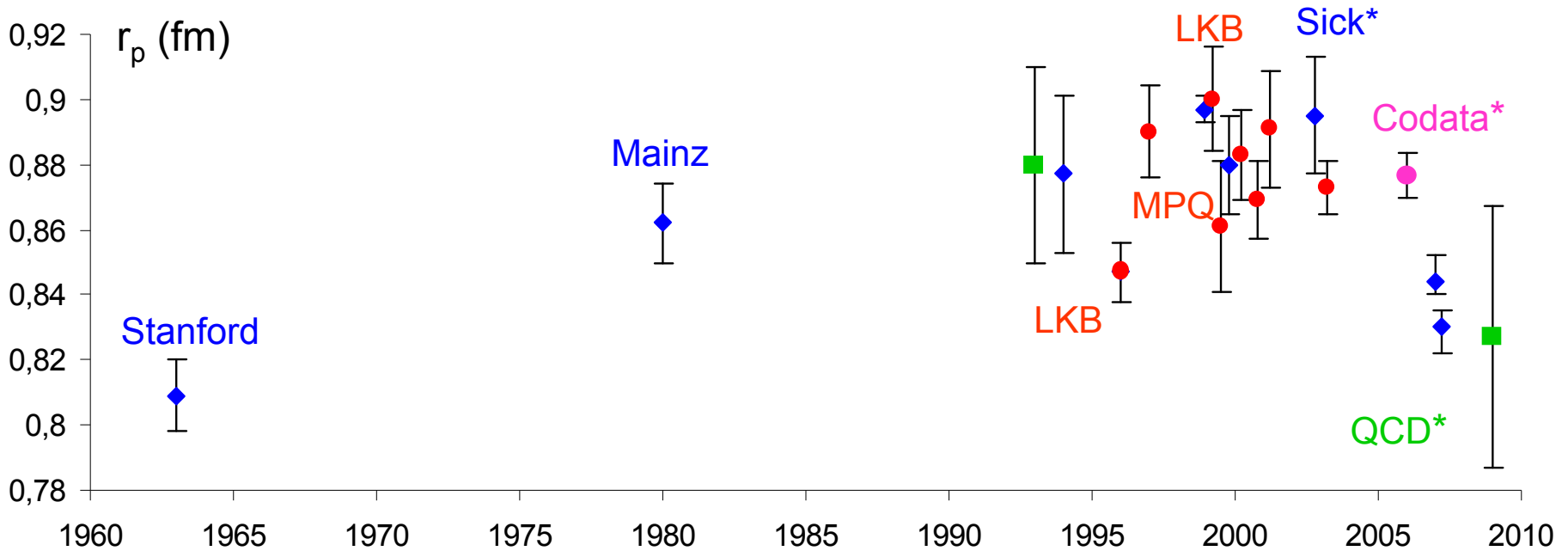
$$E(n,l,j) = hcR_\infty f(\alpha, m_e/m_p, n,l,j) + \text{recoil} + L(n,j,r_p) \approx \frac{R_\infty}{n^2} + L(n,r_p)$$

}	MPQ Garching	$\nu(1S - 2S) = (1 - \frac{1}{4})R_\infty + L(1S) - L(2S)$
	LKB Paris	$\nu(2S - 8S) = (\frac{1}{4} - \frac{1}{64})R_\infty + L(2S) - L(8S)$
	S. Karshenboim K. Pachucki	$L(1S) - 8L(2S) = \text{precisely calculated}$

Linear combinations $\rightarrow R_\infty, L^{\text{exp}}(1S)$

$$L^{\text{exp}}(1S) = 8172.840 (19) \text{ kHz} + \text{QED} \rightarrow r_p (1\%)$$

Previous determinations of the “proton radius”



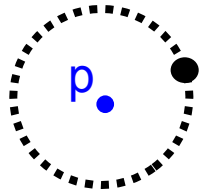
* Hydrogen data(1S-2S and 2S-nS/nD) + QED re-evaluation : 0,8760 (78) fm

* Electron scattering (all meas+re-analysis): 0,895 (18) fm

* QCD (lattice cal.) : 0,827 (40) fm

Muonic hydrogen spectroscopy

Exotic atom

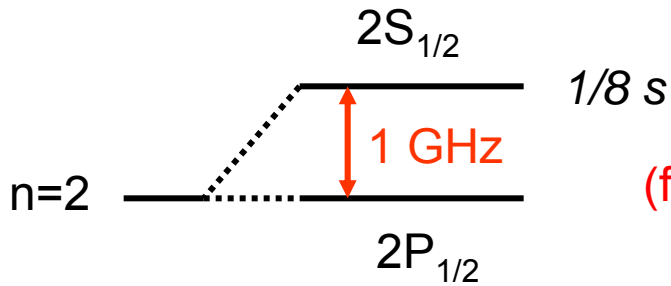


$$\frac{m_{\mu}}{m_e} = 207$$

radius : $\sim a_0/200$

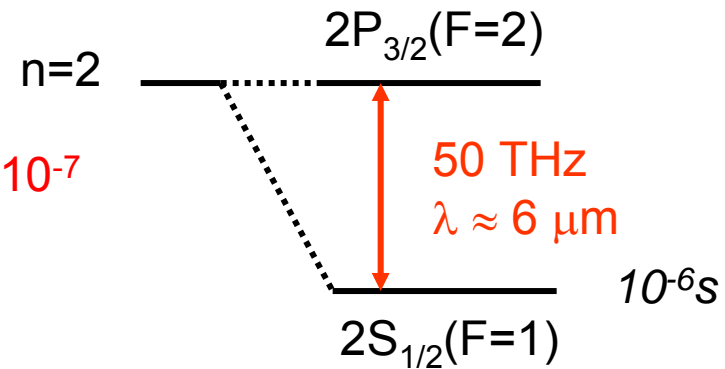
$$E_{NS} = \frac{4}{3} \left(\frac{m_r}{m_e} \right)^3 \frac{R_{\infty} hc}{n^3} \left(\frac{r_p}{a_0} \right)^2 \delta_{l0}$$

electronic hydrogen : e-p



$$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \approx 10^{-7}$$

muonic hydrogen : μ -p



$$\Delta E_{2S-2P} = 50.7724(12) - 1.2637 r_p^2 - 0.0084 r_p^3 = 49.8083 (149) \text{ THz}$$

with $r_p = 0.8760(68) \text{ fm}$

Outline

1. Introduction

- proton properties
- experimental proton size determinations
- muonic hydrogen

2. Experiment

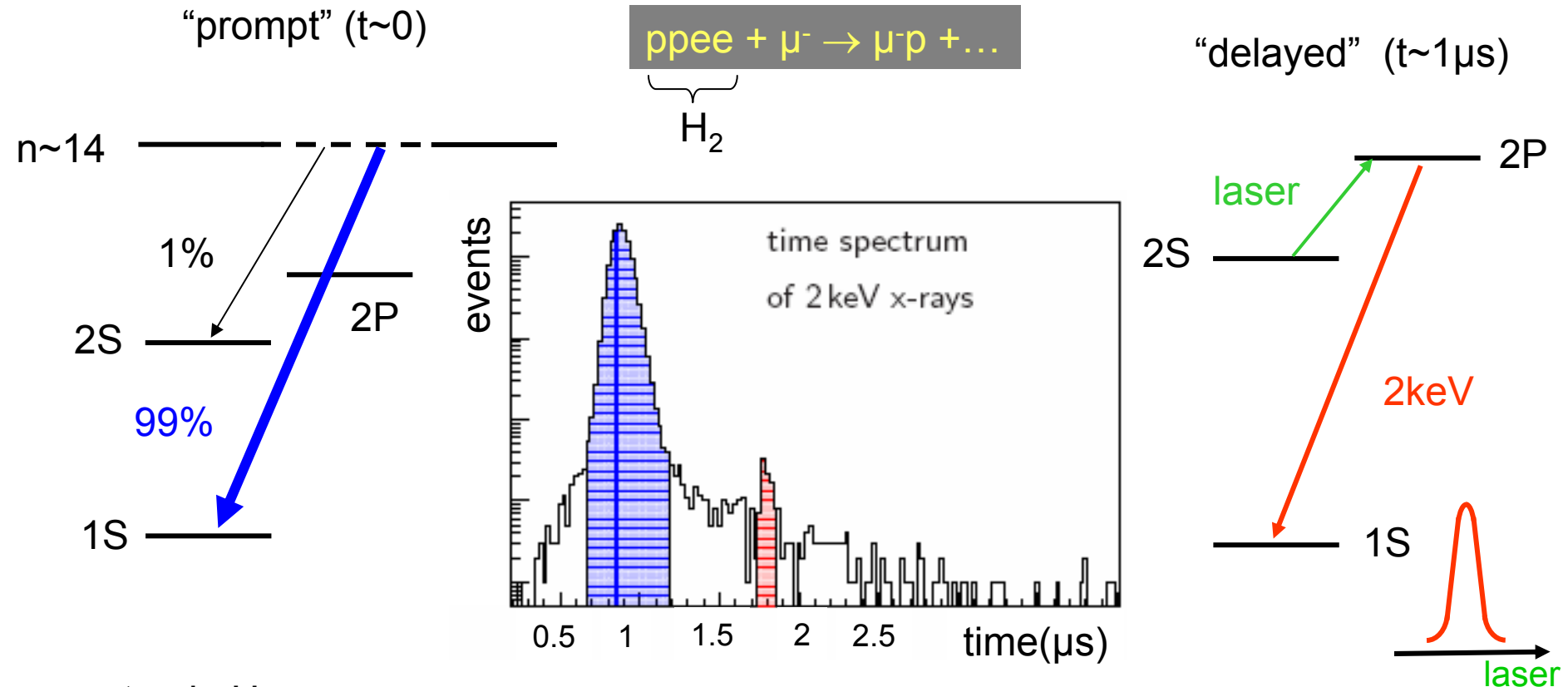
- muonic hydrogen source
- laser system
- analysis

3. Results

- signal
- proton radius from muonic hydrogen spectroscopy
- outcomes

4. Future

Principle of the experiment



μ^- stop in H_2 gas
 $\Rightarrow \mu\text{p}^*$ atoms formed ($n \sim 14$)

99%: cascade to 1S emitting prompt $\text{K}\alpha, \text{K}\beta, \dots$

1%: long lived 2S state ($\tau \sim 1 \mu\text{s}$ at 1mbar)

Fire laser ($\lambda \sim 6 \mu\text{m}$, $\Delta E \sim 0.2 \text{eV}$)
 \Rightarrow induce $\mu\text{p}(2\text{S}-2\text{P})$

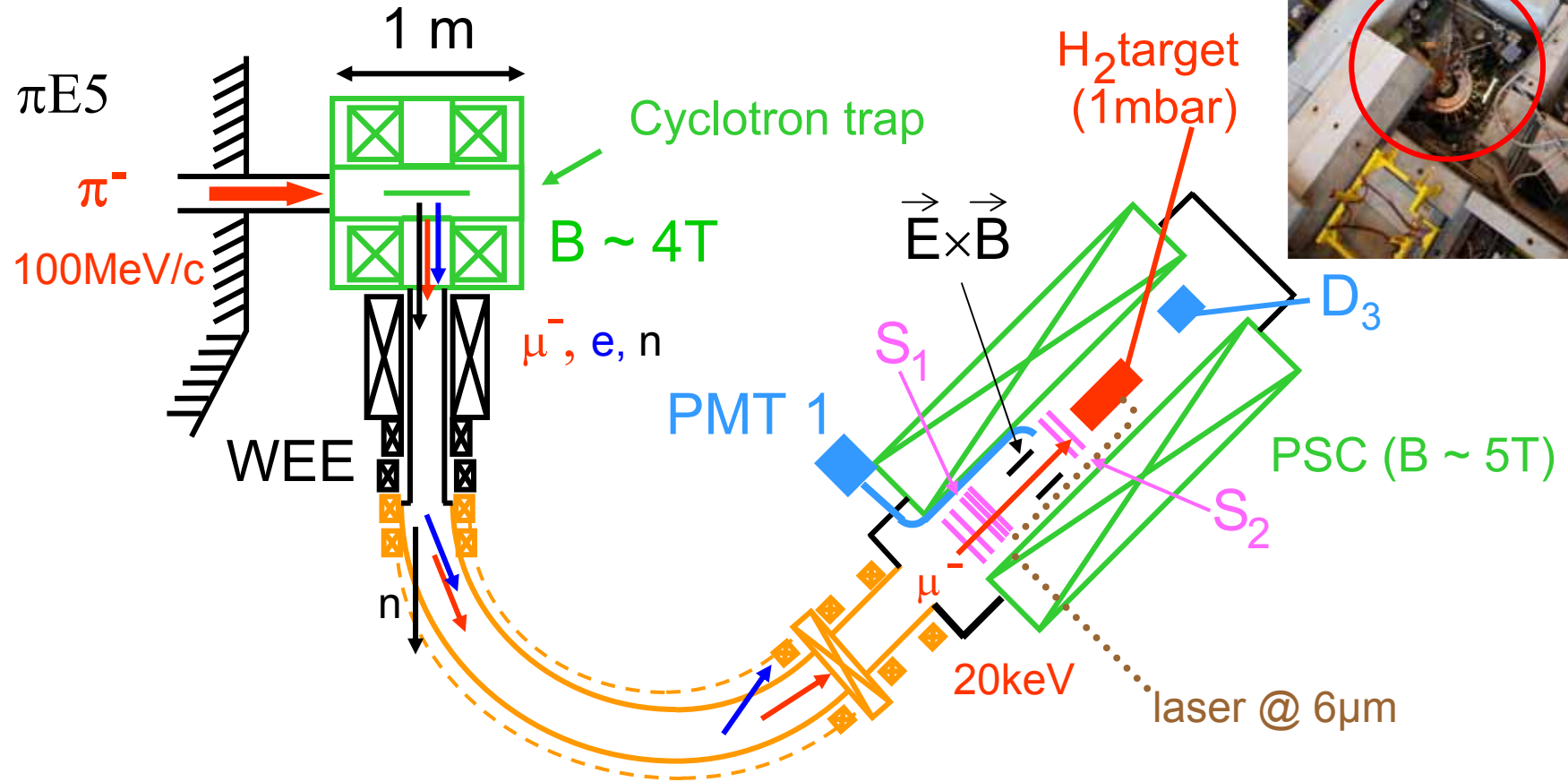
\Rightarrow observe delayed $\text{K}\alpha$ x-rays

\Rightarrow normalize $\frac{\text{delayed } \text{K}\alpha}{\text{prompt } \text{K}\alpha}$ x-rays

muonic hydrogen source

p (500MeV, 2mA) \rightarrow carbon target \rightarrow




$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad (300 \mu\text{/s})$$



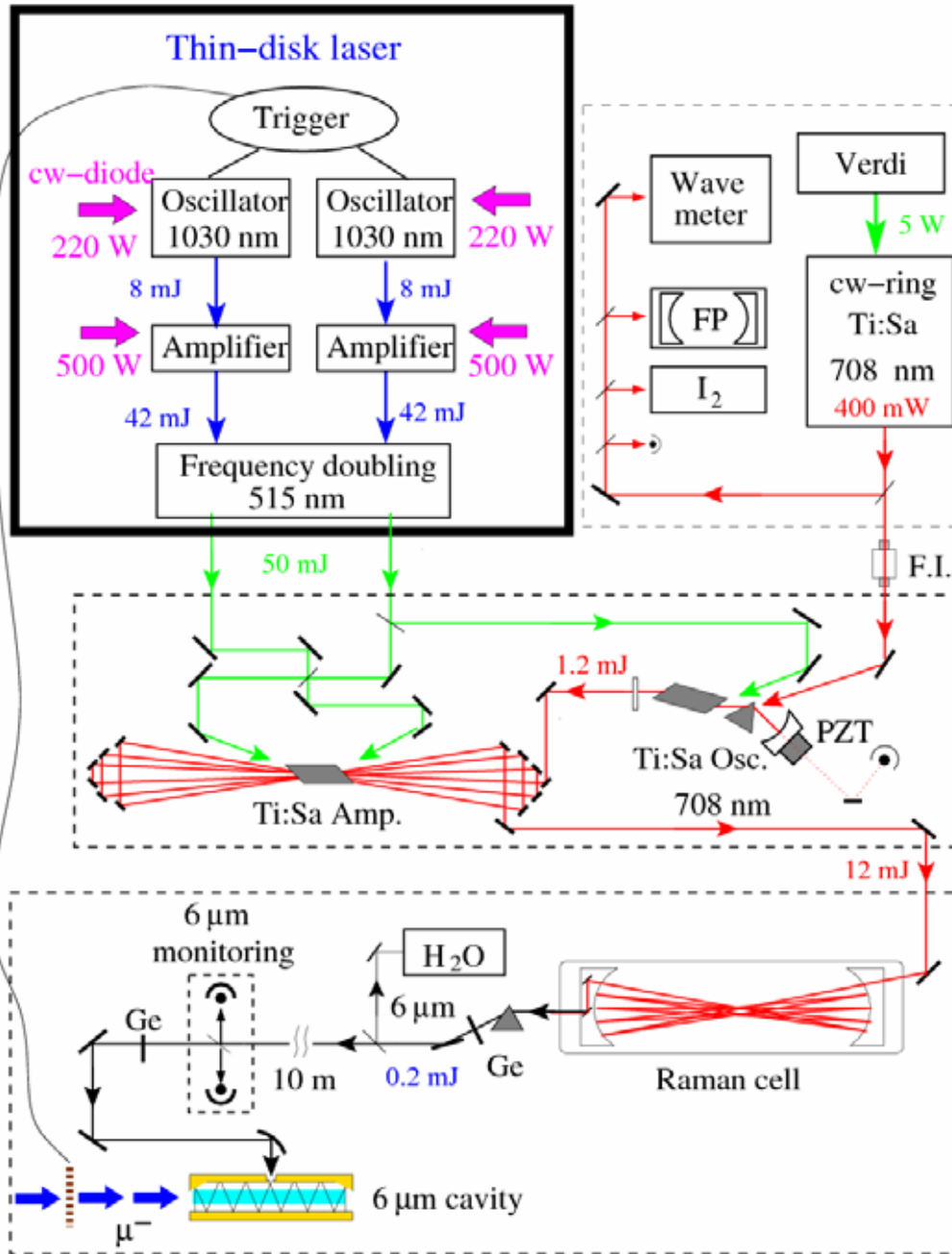
Muon Extraction Channel

$S_1 S_2 \rightarrow$ T.O.F. \rightarrow laser trigger

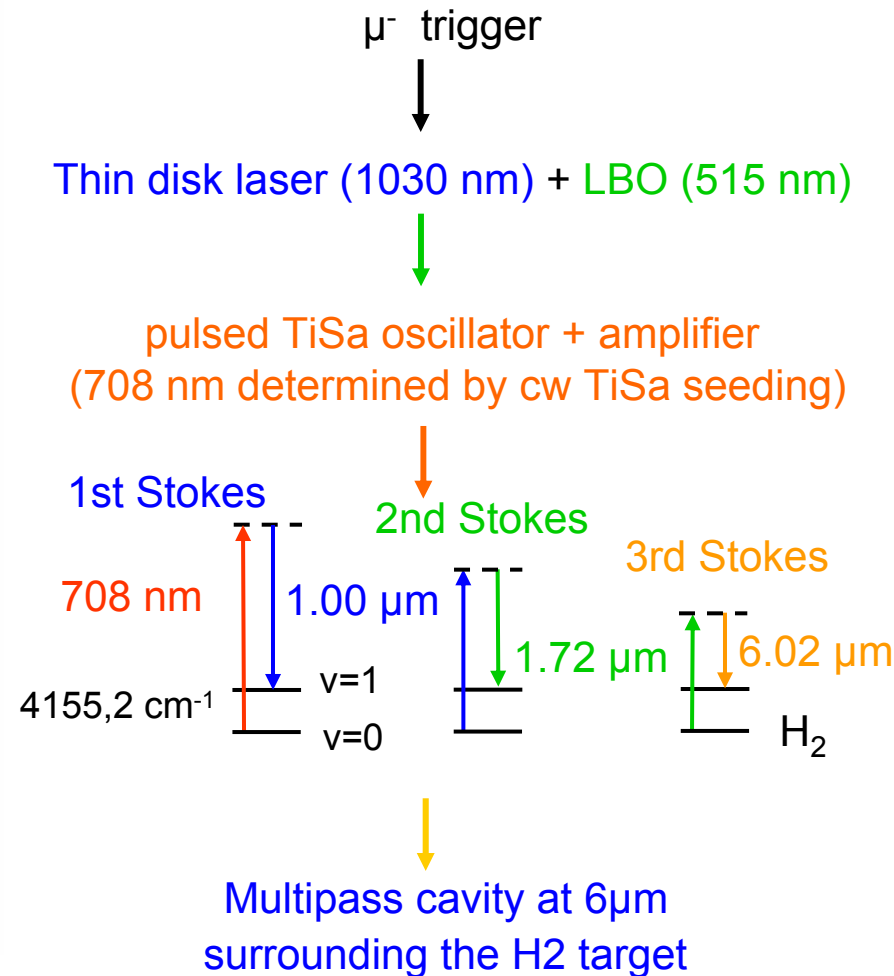
muon life in CREMA experiment

-  pion π^-
-  muon μ^-
-  photon from scintillator

Laser chain



- Each single muon triggers the laser system (random trigger)
- 2S lifetime $\sim 1\mu\text{s}$ \rightarrow **short laser delay (disk laser)**
- **6 μm tunable** laser pulse (0.2mJ)



X-rays analysis → noise rejection

Example : FP 900 - 11 hrs meas.

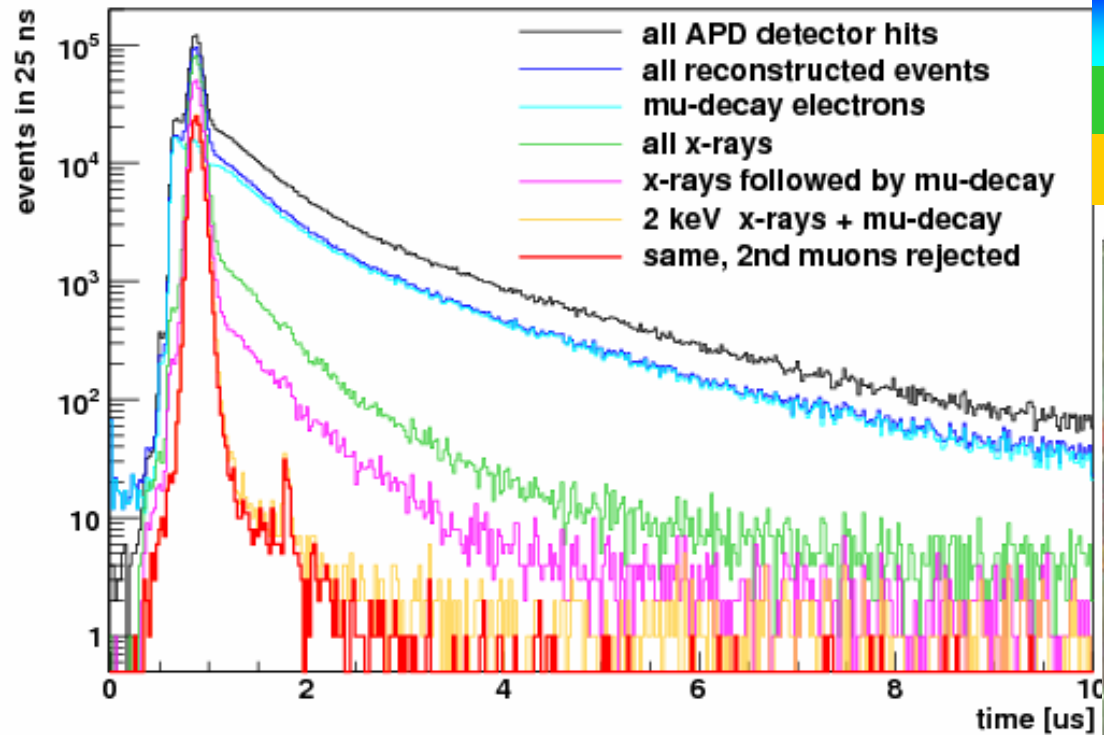
- 400 μ^- /s
- 240 laser shot/s
- 860 000 laser shot/hour
- 1.56 million detector clicks
- 19600 clicks in the laser region
- expected 2-3 laser induced events/hour !

time signature in LAAPD

- photon < 10keV → 1 shot in the LAAPD
- e^- in B = 5T → many counts in detectors

energy signature in LAAPD

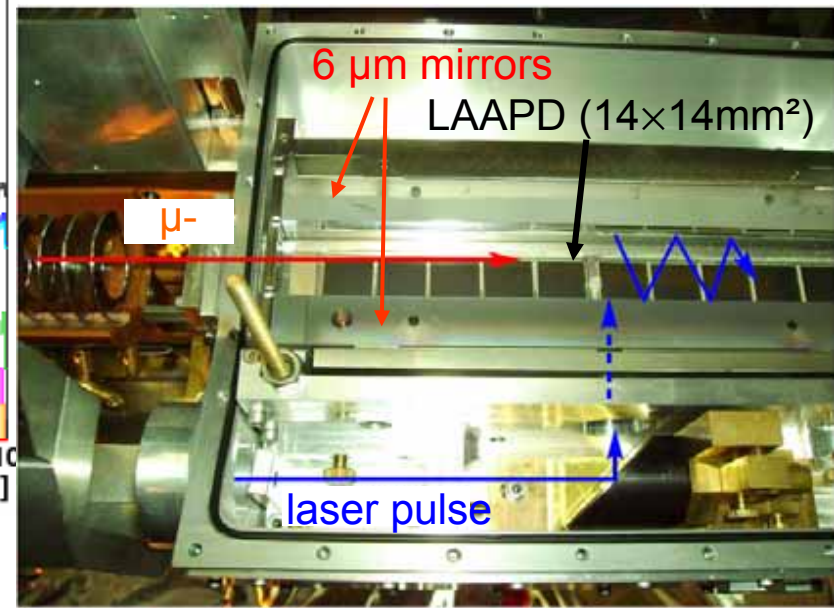
- $E > 8\text{keV} \Leftrightarrow$ electron
- $1\text{keV} < E < 8\text{keV} \Leftrightarrow$ X ray
- $E < 1\text{keV} \Leftrightarrow$ neutron



$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$

x rays multiplicity 1

LAAPD energy resolution



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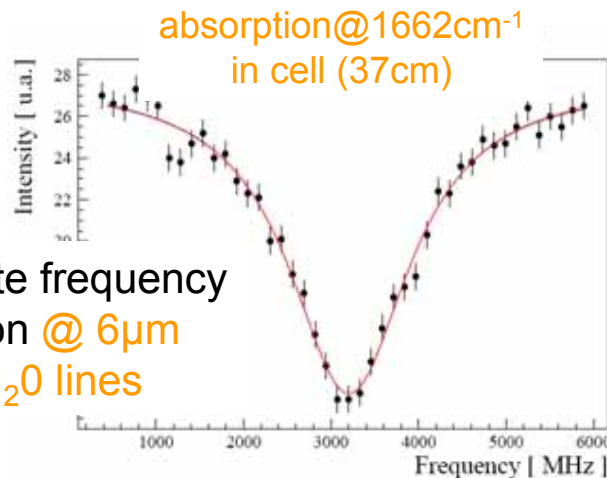
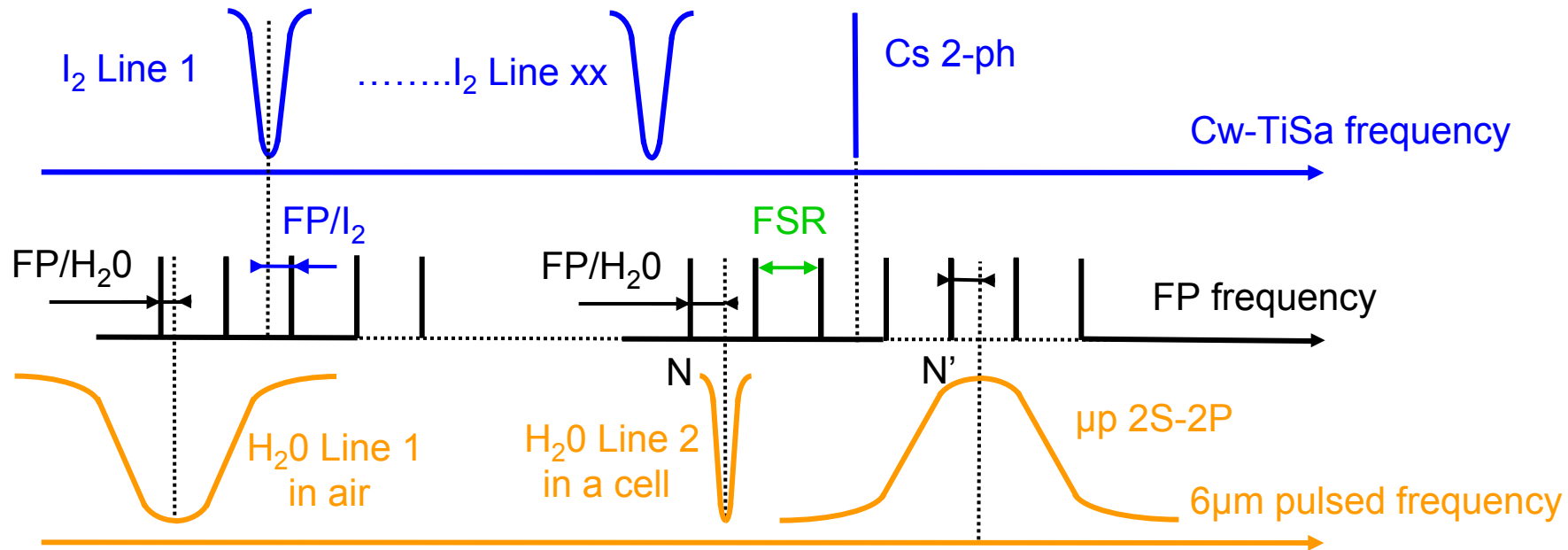
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Laser chain : frequency calibration

FSR measured/controlled in cw with I₂ (1 ph abs), Cs (2 ph fluo), Rb (2 ph fluo), lines



$$\nu(\mu\text{p}:2\text{S}-2\text{P}) = \nu(\text{H}_2\text{O Line 2}) + (N-N') \text{ FSR}$$

FP absolute frequency calibration @ 6µm with H₂O lines

$\mu\text{p } (2S_{1/2}(F=1)-2P_{3/2}(F=2))$: uncertainty budget

Statistics

- uncertainty on position (fit) 700 MHz ($\sim 4\%$ of Γ_{nat})

$$\Delta\nu_{\text{experimental}} = 20 (1) \text{ GHz} \quad (\Gamma_{\text{nat}} = 18.6 \text{ GHz})$$

Sources :

- Laser frequency (H_2O calibration) 300 MHz
- AC and DC stark shift < 1 MHz
- Zeeman shift (5 Telsa) < 30 MHz
- Doppler shift < 1 MHz
- Collisional shift 2 MHz

TOTAL UNCERTAINTY ON FREQUENCY

762 MHz

Broadening :

- 6 μm laser line width ~ 2 GHz
- Doppler Broadening < 1 GHz
- Collisional broadening 2.4 MHz

$$\nu (\mu\text{p} : 2S_{1/2}(F=1)- 2P_{3/2}(F=2)) = 49\,881.88 (76) \text{ GHz} \quad (1.6 \times 10^{-5})$$

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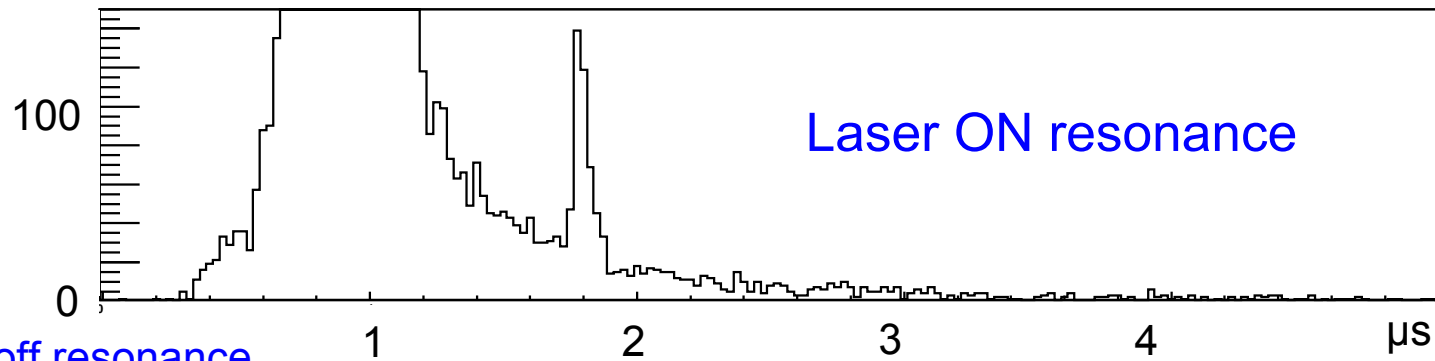
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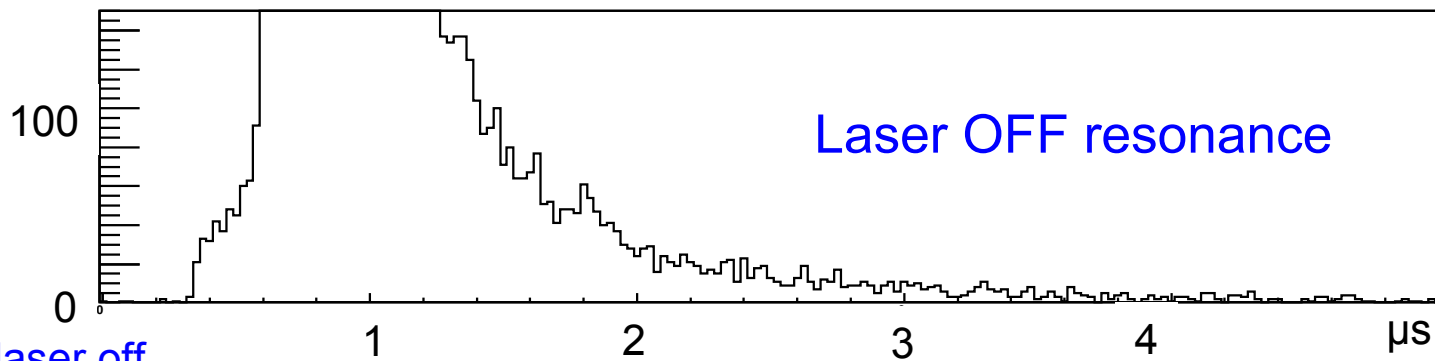
4. Future

muonic hydrogen: $2S_{1/2}(F=1)$ - $2P_{3/2}(F=2)$ time spectra

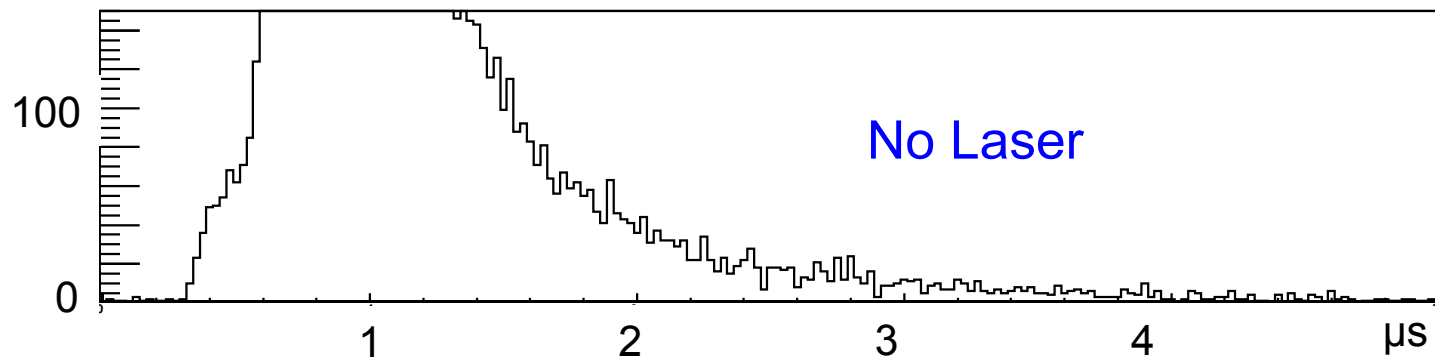
summed on resonance



summed off resonance



summed laser off



Prospect : muonic helium spectroscopy at PSI

2013....→ muonic helium spectroscopy

- μHe^+ spectroscopy + He^+ spectroscopy → QED test ($Z\alpha$)
- test of the spectroscopy of muonic system

Up date comparison

New electron-proton scattering : Phys Rev Lett 105, 242001 (2010)

Model independent analysis of proton structure...:Phys Rev Lett 107, 160402 (2011)

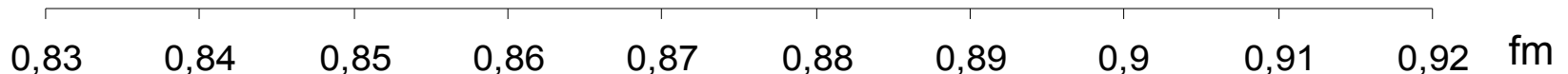
$$\nu (\mu\text{p} : 2S_{1/2}(F=1) - 2P_{3/2}(F=2)) = 49\,881.88 (76) \text{ GHz}$$
$$\nu_{\text{theo}} = 50772.43 (967) - 1263.69 r_p^2 + 8.39 r_p^3 \text{ GHz}$$
$$r_p = 0.84184(36)(458) \text{ fm}$$

electron-proton scattering re-analysis |-----●-----| 0,895 (18) fm

H/D spectroscopy + QED |-----●-----| 0.8760 (78) fm

New electron-proton scattering |-----●-----| 0.879 (8) fm

|-----●-----| $\mu\text{p} (2S_{1/2}(F=1) - 2P_{3/2}(F=2))$ spectroscopy + QED 0.8418 (46) fm

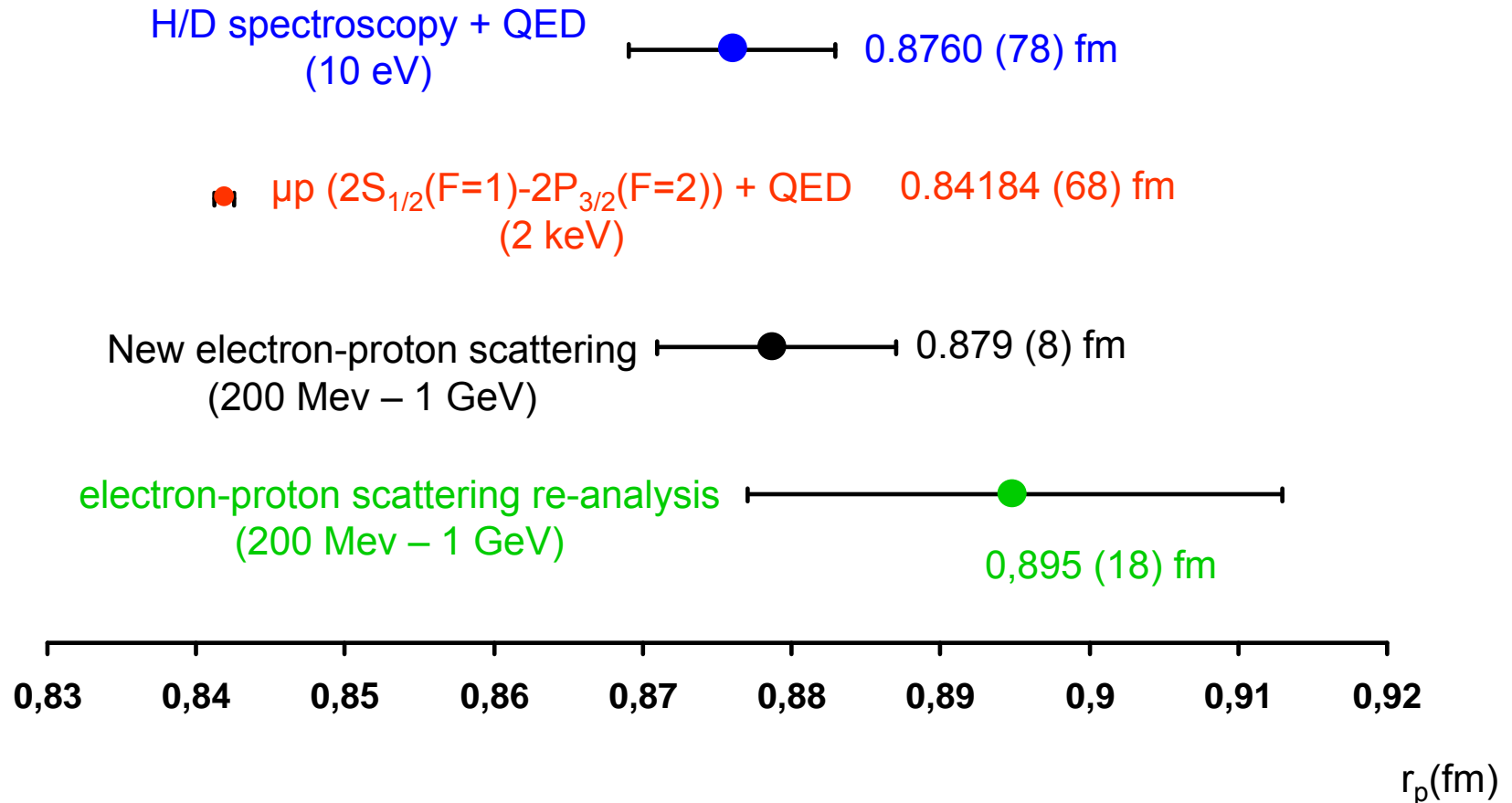


Comparison with other best determinations

$$v_{\text{theo}} = f(R_{\infty}, \text{QED}, r_p) = 50772.43(1.18) - 1263.69 r_p^2 + 8.39 r_p^3 \text{ GHz}$$

$$v (\mu p : 2S_{1/2}(F=1) - 2P_{3/2}(F=2)) = 49\,881.88 (76) \text{ GHz}$$

$$\Rightarrow r_p = 0.84184(36)(56) \text{ fm}$$



New electron-proton scattering : Phys Rev Lett 105, 242001 (2010)

Muonic hydrogen : *Nature* 466, 213-216 (2010)