# Du micromètre à l'attomètre : une détermination optique de la taille du proton

François Nez

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Submitted on 10 Jan 2012

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# Du micromètre à l'attomètre : une détermination optique de la taille du proton

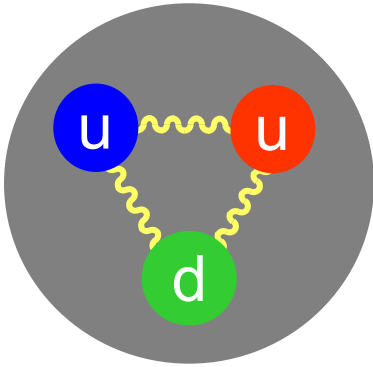
F.D Amaro, A. Antognini, F.Biraben, J.M.R. Cardoso, D.S. Covita, A. Dax, S. Dhawan, L.M.P. Fernandes, A. Giesen, T. Graf, T.W. Hänsch, P. Indelicato, L.Julien, C.-Y. Kao, P.E. Knowles, F. Kottmann, J.A.M. Lopes, E. Le Bigot, Y.-W. Liu, L. Ludhova, C.M.B. Monteiro, F. Mulhauser, T. Nebel, F. Nez, R. Pohl, P. Rabinowitz, J.M.F. dos Santos, L.A. Schaller, K. Schuhmann, C. Schwob, D. Taqqu, J.F.C.A. Veloso

http://muhy.web.psi.ch http://www.lkb.ens.fr/-Metrologie-Quantique-

# Proton

## Structure

2 quarks up (2/3 e) + 1 quark down (-1/3 e) + strong interaction (gluons)



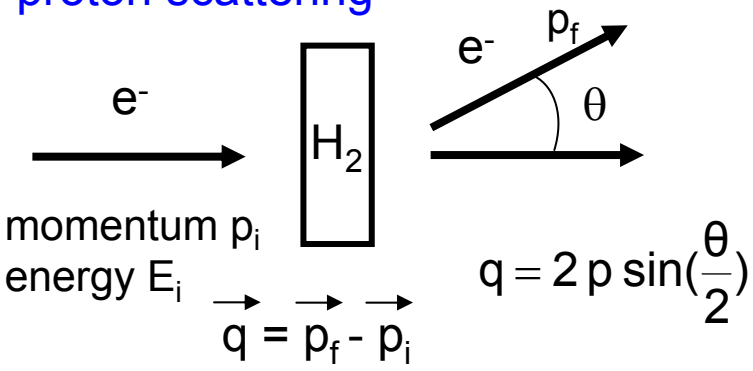

"proton size" $\lambda_c = \frac{h}{m_p c} \approx 1.3\,\text{fm}$

## Proton charge radius : $r_p \sim 0.8$ fm

$$r_p = \sqrt{\langle r^2 \rangle} \qquad \langle r^2 \rangle = \frac{\int r^2 \rho(r)\, d^3r}{\int \rho(r)\, d^3r}$$

$\rho(r)$ : proton charge distribution

## Electron-proton scattering



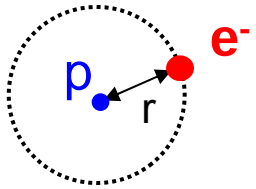

$$q = 2p \sin\left(\frac{\theta}{2}\right)$$

$$\frac{d\sigma}{d\Omega}(E_i,\theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i,\theta)\right]}_{\text{Rutherford}} G(q^2)$$

$$G(q^2) = \int d^3r\, e^{iqr} \frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6} q^2$$

$r_p$ deduced from extrapolation $q \to 0$ with $\rho(r)$ unknown…

# hydrogen spectroscopy



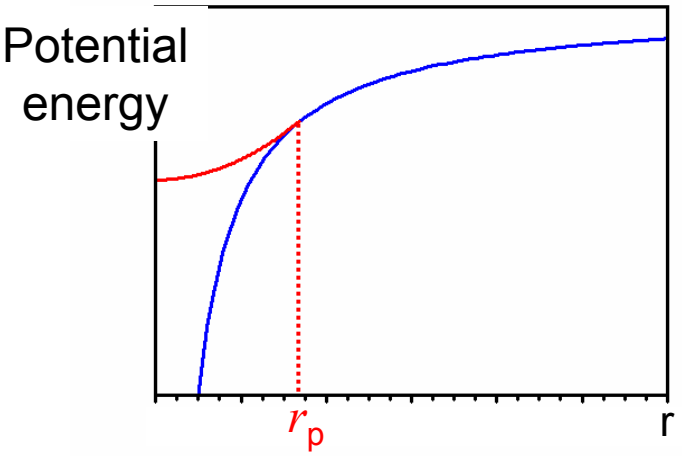

$E(n,l,j) = \text{Dirac} + \text{recoil} + L(n,j)$

$hcR_\infty f(\alpha, m_e/m_p, n,l,j)$

exact

not well known

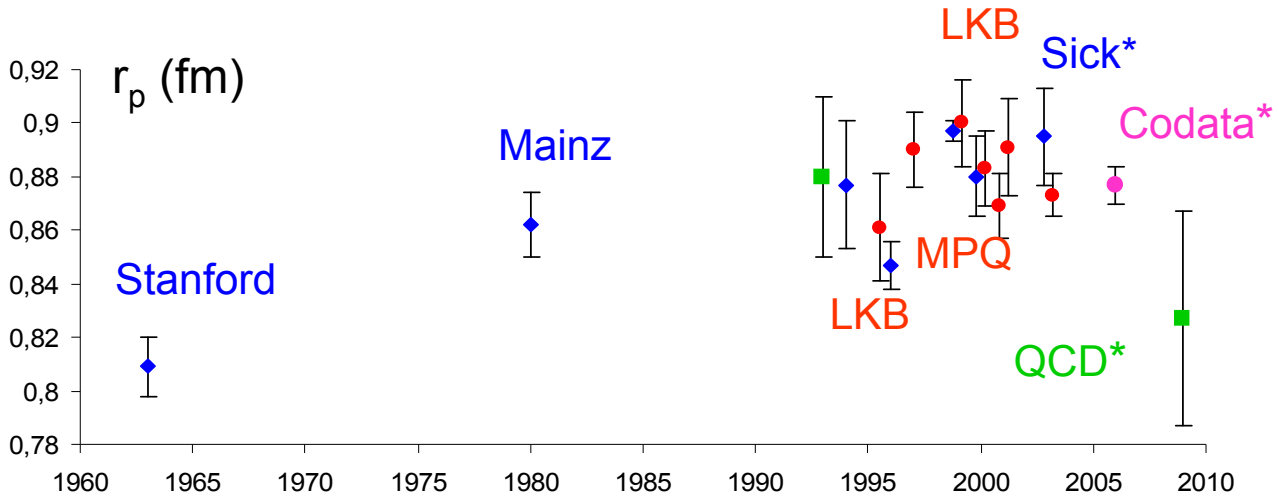
- QED corrections ($1/n^3$)
- relativistic recoil
- charge radius of the proton ($1/n^3$)




$f_{1S\text{-}2S}$= 2 466 061 413 187 035 (10) Hz

PRL 107 203001 (2011)

# Previous determinations of the "proton radius"

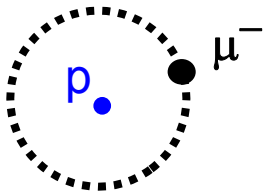



* Hydrogen data(1S-2S and 2S-nS/nD) + QED re-evaluation : 0,8760 (78) fm

* Electron scattering (all meas+re-analysis): 0,895 (18) fm $f_{1S\text{-}2S}$= 2 466 061 413 235 502 Hz

* QCD (lattice cal.) : 0,827 (40) fm

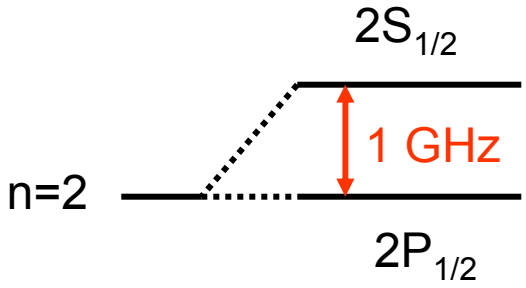
# Muonic hydrogen spectroscopy

Exotic atom 


$\frac{m_\mu}{m_e} = 207$

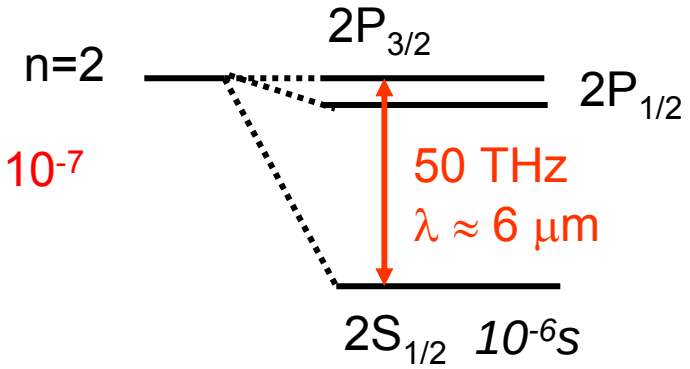
radius : ~ $a_0/200$

electronic hydrogen : e-p




$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \approx 10^{-7}$

muonic hydrogen : μ-p




Relative contribution due to $r_p$ to 2S-2P

$1.4\times10^{-4}$

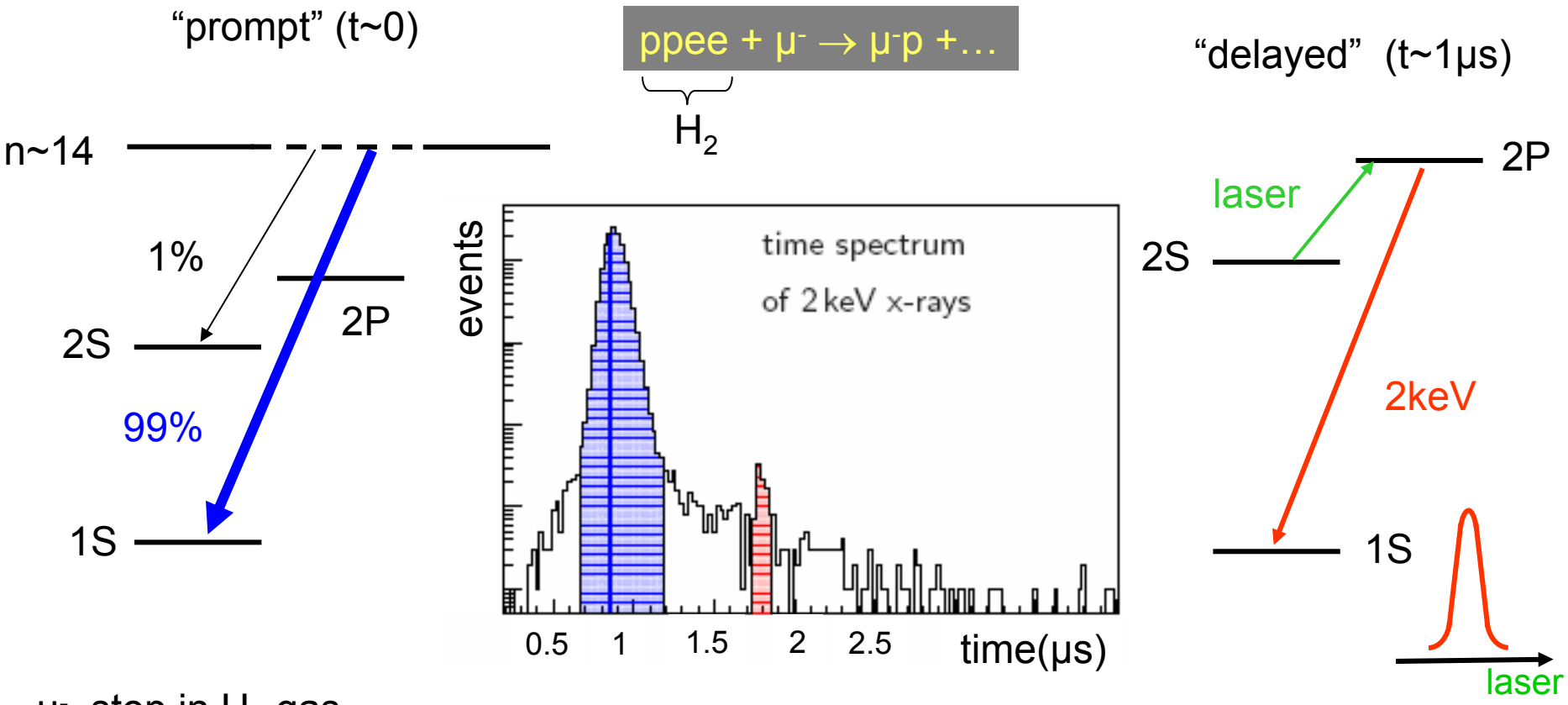
$2\times10^{-2}$

Predicted frequency of muonic hydrogen 2S-2P transition (2002)

$\Delta E_{2S\text{-}2P} = 50.7702(12) - 1.2634\ r_p^{\,2} - 0.0084\ r_p^{\,3} = 49.8063\ (149)\text{THz}$

with $r_p$=0.8760(68)fm

# Principle of the experiment




$\mu^-$ stop in $H_2$ gas
$\Rightarrow$ $\mu p^*$ atoms formed (n~14)

99%: cascade to 1S emitting prompt K$\alpha$,K$\beta$,…

1%: long lived 2S state ($\tau$ ~ 1μs at 1mbar)

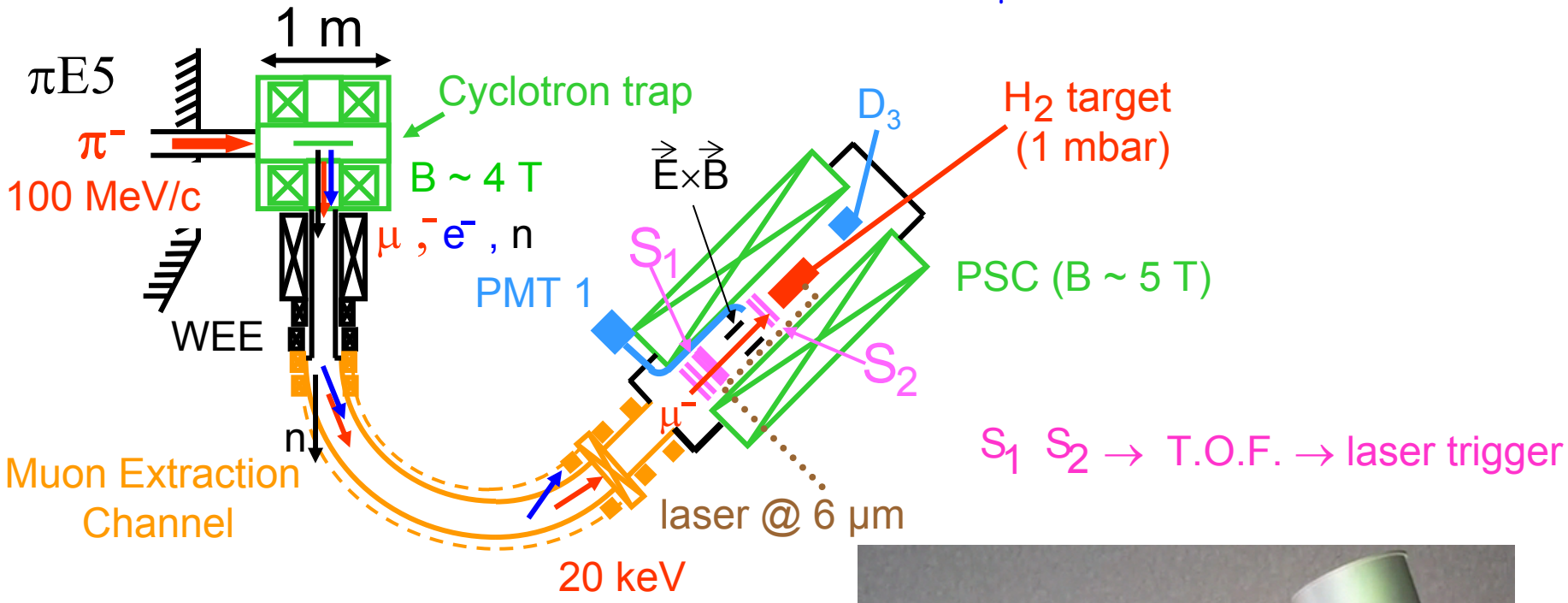
Fire laser ($\lambda$~6 μm, $\Delta$E~0.2 eV)
$\Rightarrow$ induce μp(2S-2P)

$\Rightarrow$ observe delayed K$\alpha$ x-rays

$\Rightarrow$ normalize $\frac{\text{delayed K}\alpha}{\text{prompt K}\alpha}$ x-rays

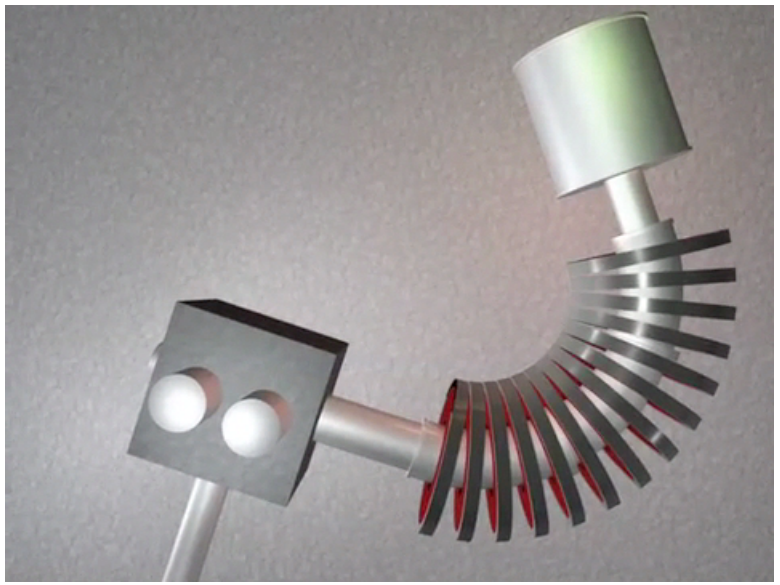
# Muonic hydrogen source

p (500 Mev, 2 mA) → carbon target → $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ (300 $\mu^-$/s)




muon life in CREMA experiment

- pion $\pi^-$
- muon $\mu^-$
- photon from scintillator

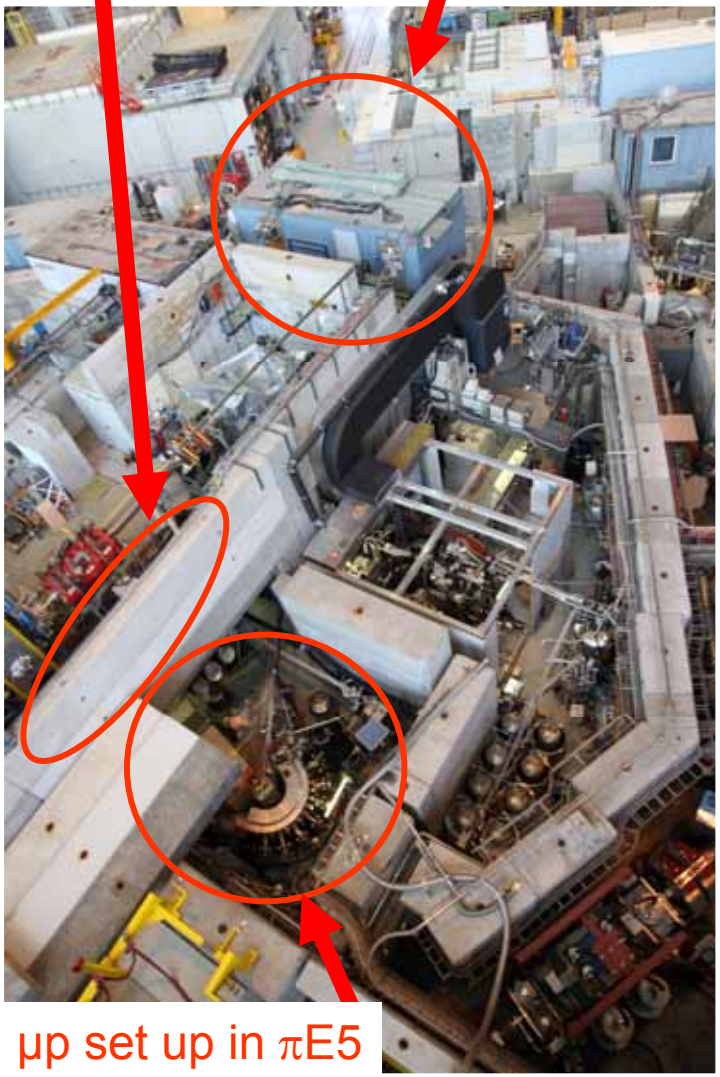


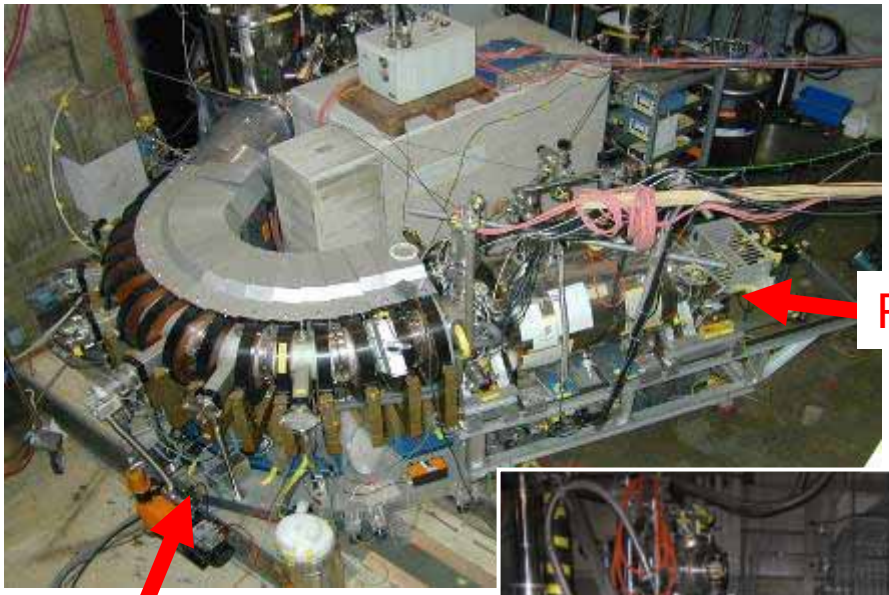
# muon beam apparatus

$\pi^- \rightarrow \mu^- + \nu_\mu$

laser hut below concrete blocks

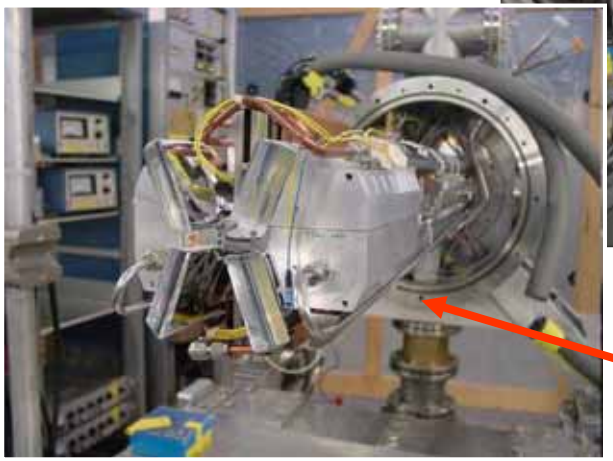
counting room





PSC solenoid,

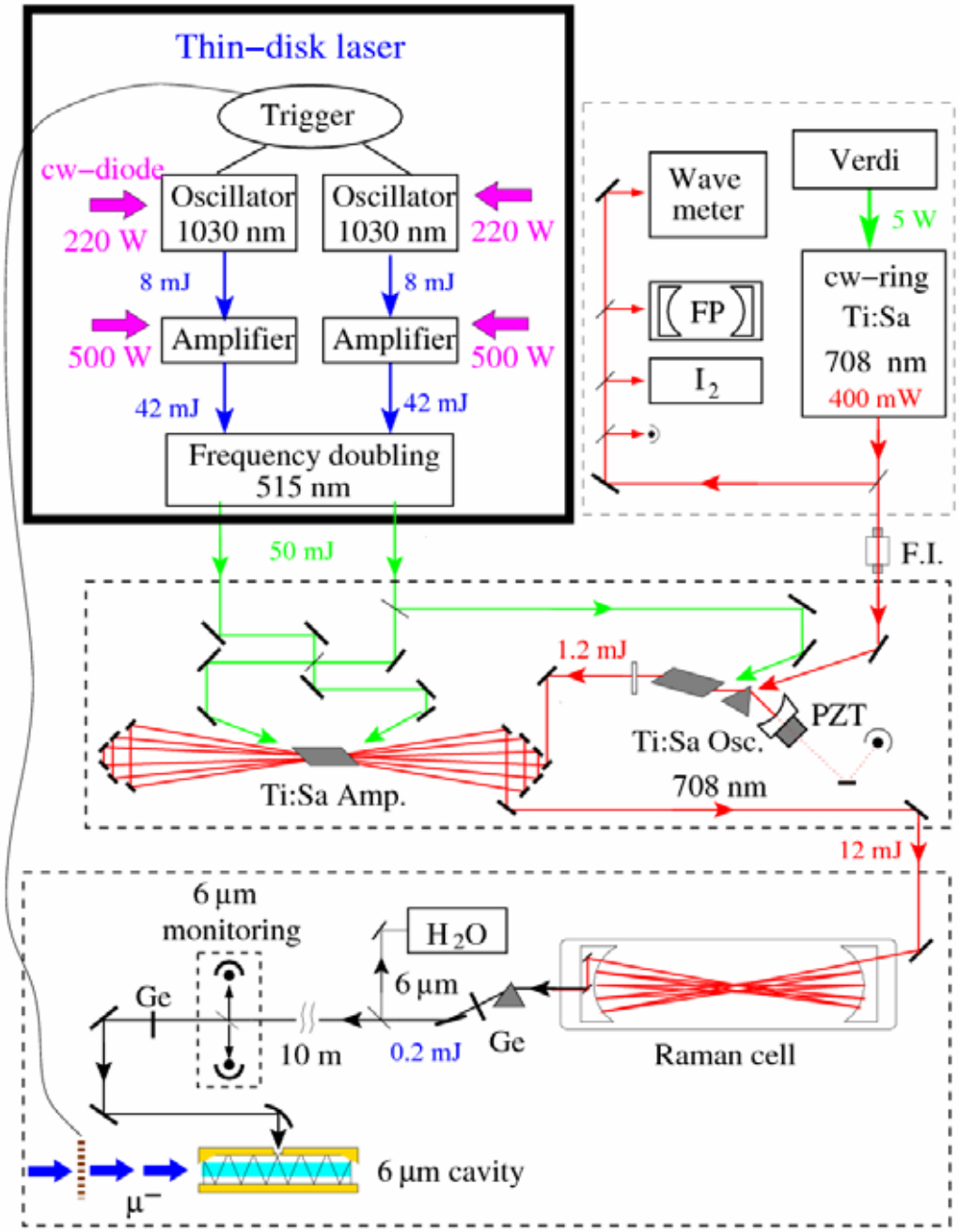
Muon extraction channel



H2 target, laser cavity, detectors

μp set up in πE5

# Laser chain




- Each single muon triggers the laser system (random trigger)
- 2S lifetime ~1 µs → short laser delay (disk laser)
- 6 µm tunable laser pulse (0.2 mJ)

µ⁻ trigger

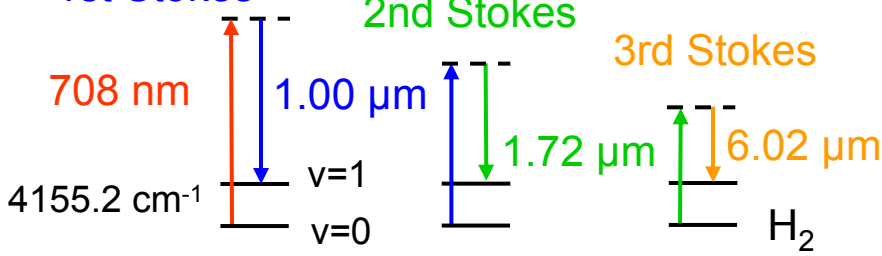
↓

Thin disk laser (1030 nm) + LBO (515 nm)

↓

pulsed TiSa oscillator + amplifier
(708 nm determined by cw TiSa seeding)

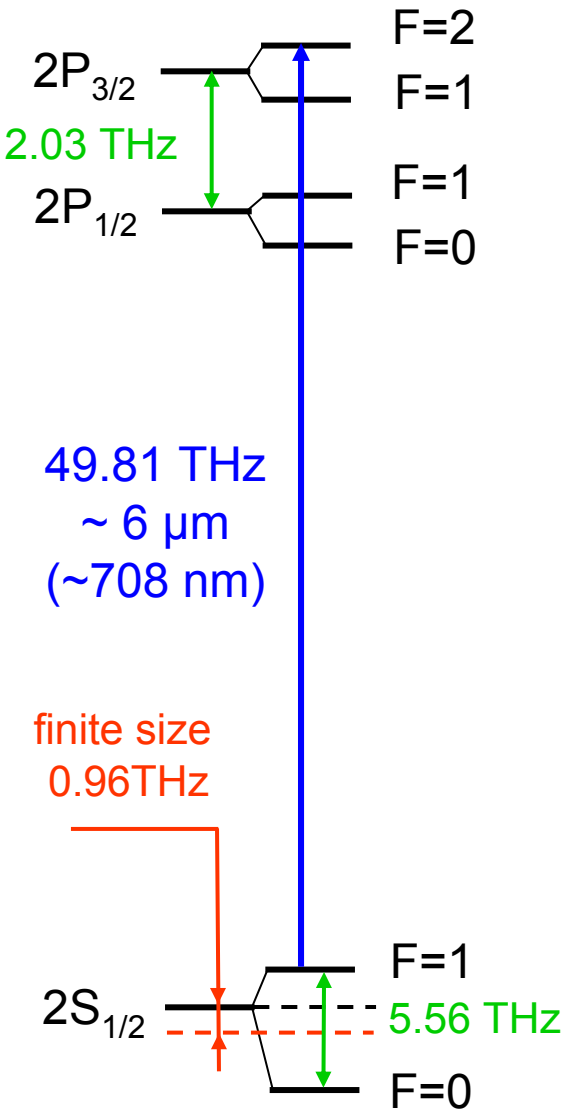
↓



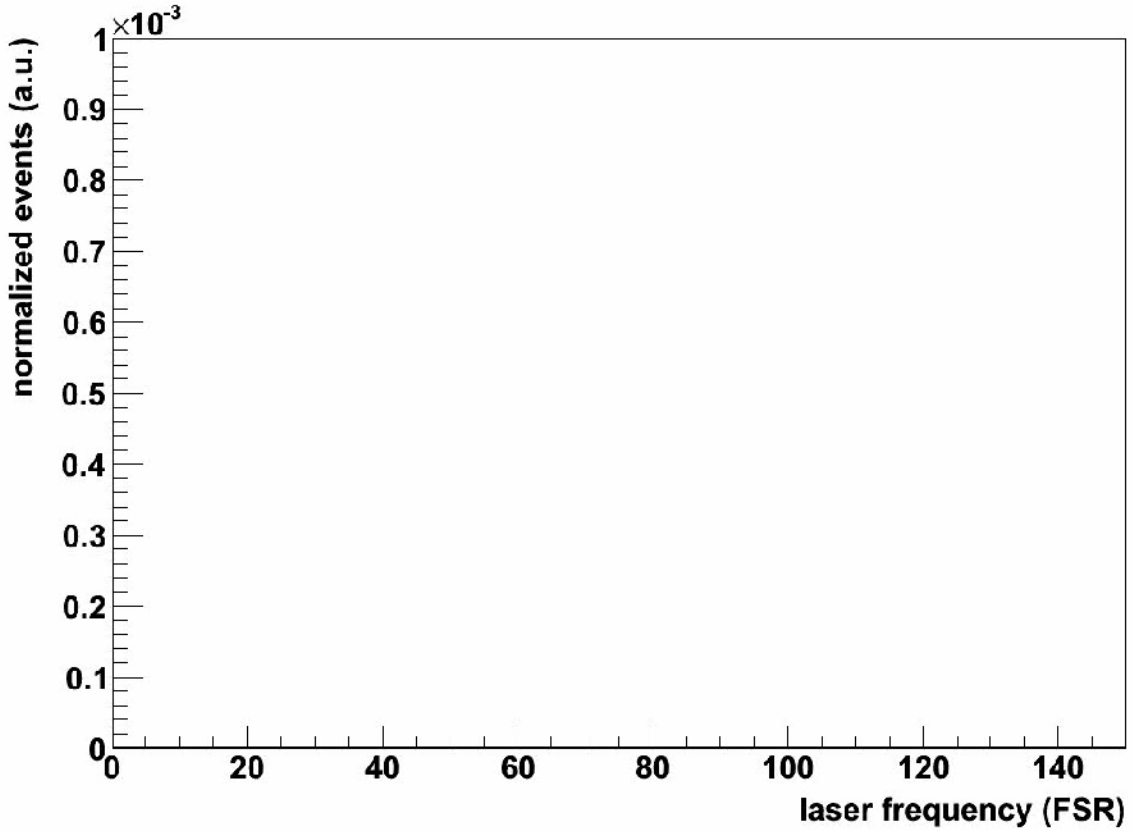

↓

Multipass cavity at 6 µm
surrounding the $H_2$ target

# muonic hydrogen : $2S_{1/2}(F=1) - 2P_{3/2}(F=2)$




- 550 events measured
- 155 backgrounds
- 31 FP fringes
- 250 hours



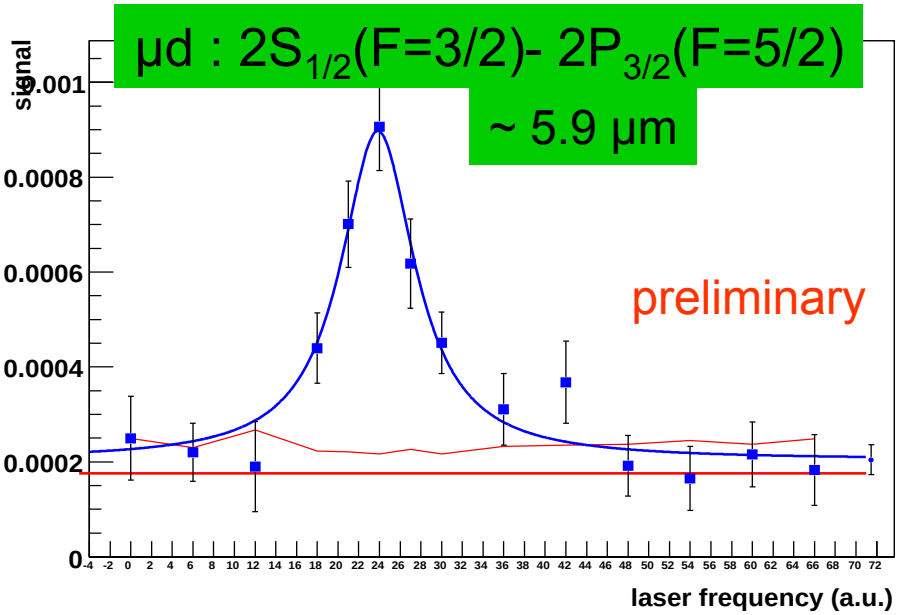

→ proton charge radius (~0.1%)

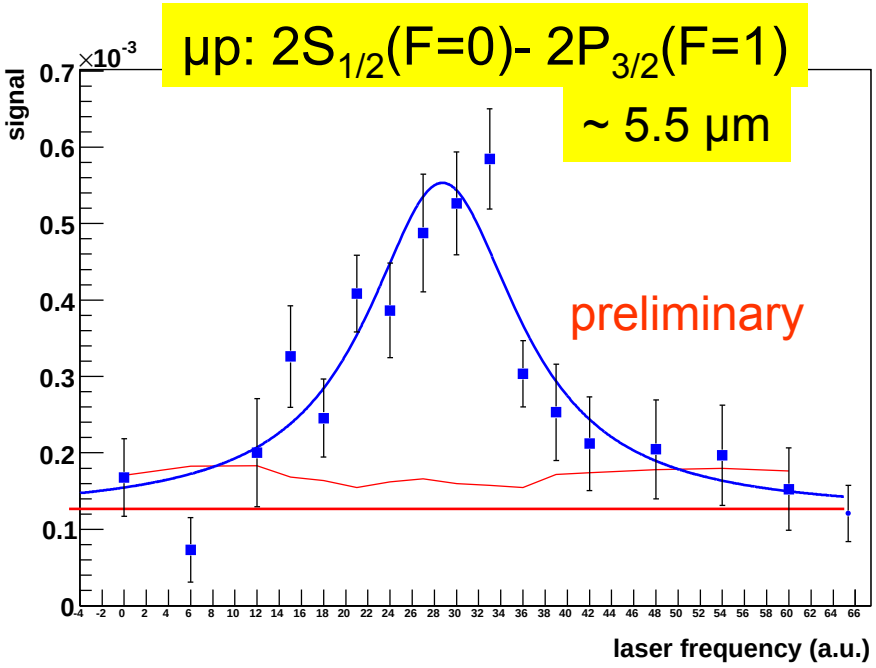
# Other lines observed in μp and μd atoms

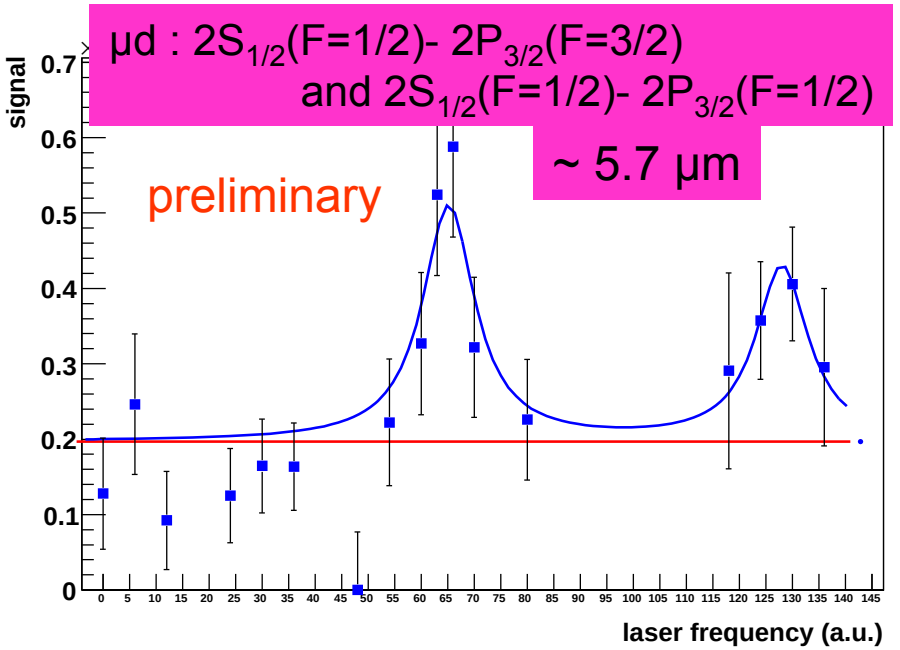
- 2nd μp line : at the correct position
hfs → Zeemach radius

- μd : not at the correct position with new rp and isotope shift
→ deuteron polarizability

- 2nd μd line : observation
→ check calculation



# Comparison with other best determinations

$\nu$ ($\mu$p : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.88 (76) GHz

$\nu_{theo}$= 50772.43(118) – 1263.69 $r_p^2$ + 8.39 $r_p^3$ GHz

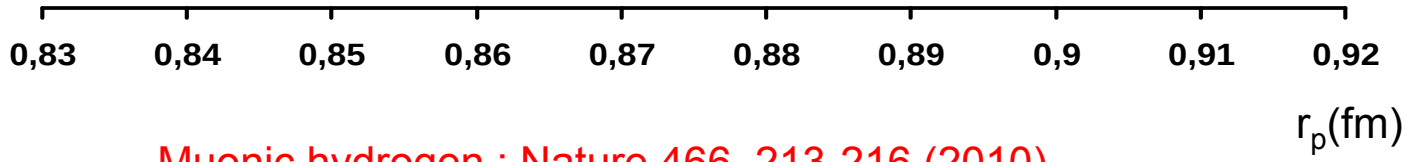
} $r_p$= 0.84184(36)(56) fm

H/D spectroscopy + QED — 0.8760 (78) fm

$\mu$p ($2S_{1/2}$-$2P_{3/2}$) + QED — 0.84184 (68) fm

New electron-proton scattering — 0.879 (8) fm

electron-proton scattering re-analysis — 0,895 (18) fm




Muonic hydrogen : Nature 466, 213-216 (2010)

New elec-prot scattering : Phys Rev Lett 105, 242001 (2010)

$f_{1S-2S}$= 2 466 061 413 187 035 (10) Hz  PRL 107 203001 (2011)

$f_{1S-2S}$= 2 466 061 413 109 090 Hz

# New $r_p$ (μp ($2S_{1/2}(F=1)$ - $2P_{3/2}(F=2)$)) : outcomes

Muonic hydrogen Lamb shift theory ? checked

Hydrogen spectroscopy, Rydberg constant $R_\infty$ ?

Codata 2006 *"The experimental and theoretical H and D data remain highly consistent."*

First measurement of 1S-3S (LKB) : 2 922 743 278 678(13) kHz. EPJ D **60** p.243 (2010)
Prediction : 2 922 743 278 671.6(1.4) kHz PRL **95**, 163003 (2005)

"New physics" ?

*"We consider exotic particles that, among leptons, couple preferentially to muons and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1, and spin-2 particles as an explanation."* PRL **106**, 153001 (2011)

Example of constrain : magnetic moment anomaly of the electron (Harvard) g-2= f(α/π)

With recoil (LKB) $v_r = \frac{\hbar k}{m(Rb)}$ $\alpha^2 = \frac{2R_\infty}{c}\frac{m(Rb)}{m_p}\frac{m_p}{m_e}\frac{h}{m(Rb)}$ PRL **106**, 080801 (2011)

Polarizability of the proton ?

Model independent analysis of proton structure…:Phys Rev Lett 107, 160402 (2011)

$\nu$ (μp : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.88 (76) GHz

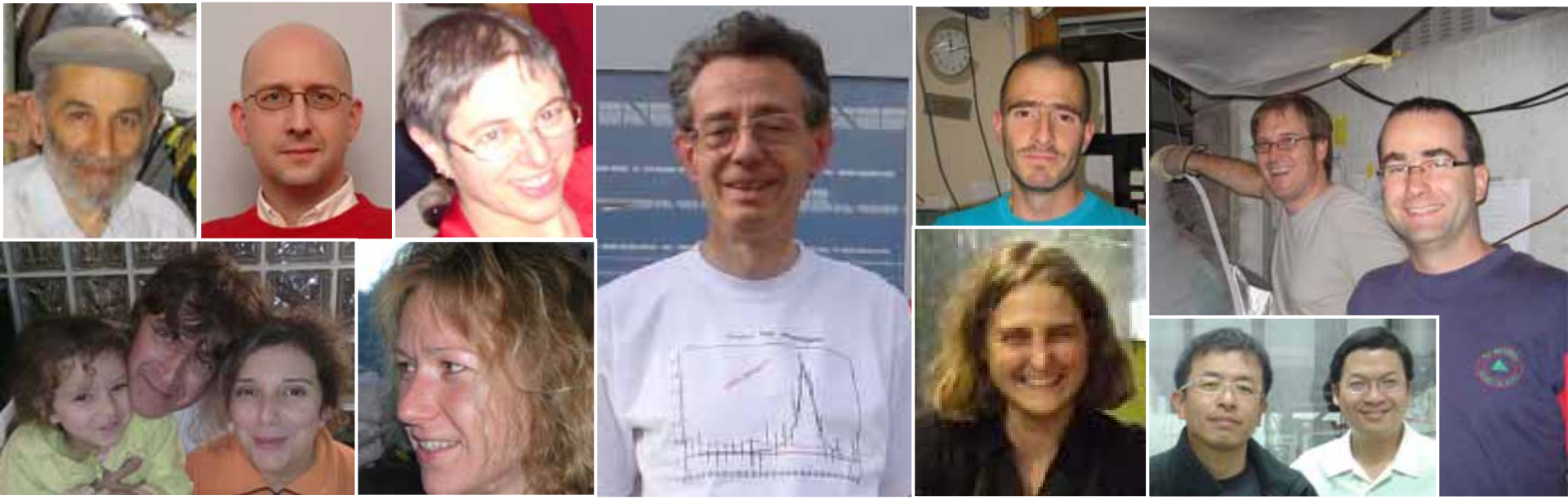
$\nu_{theo}$= 50772.43 (967) – 1263.69 $r_p^2$ + 8.39 $r_p^3$ GHz

$r_p$= 0.84184(36)(458) fm

# Hydrogen spectroscopy, QED tests, …

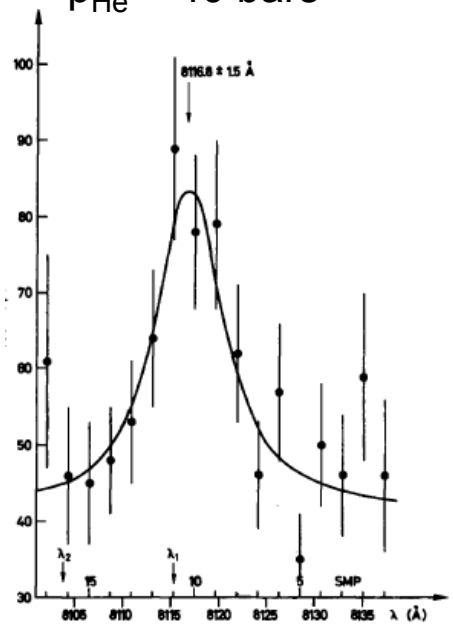
- Hydrogen Deuterium 2S-nS/nD spectroscopy at NPL,
  J.L. Flowers et al IEEE Trans. Instrum. Meas. 56, (2007) 331.
- $^{20}Ne^{9+}$ Rydberg states at NIST-Gaithersburg , U. D. Jentschura et al, PRL 100**,** 160404 (2008).
- Hydrogen 1S-2S at MPQ-Garching hydrogen cooled with magnetic field
- Hydrogen 1S-3S picosecond spectroscopy at MPQ-Garching E. Peters on going PhD
- Hydrogen 1S-3S cw-spectroscopy at LKB-Paris S.Galtier on going PhD
- $He^+$ spectroscopy femtosecond spectroscopy at MPQ-Garching
  M. Herrmann et al, PRA **79**, 052505 2009.
- Muonic helium ($\mu^4He^+$) spectroscopy at PSI-Villigen CREMA collaboration Proposal 2010.
- Electron-proton scattering MAMI/A1 collaboration at Mainz
  J. C. Bernauer et al., Can. J. Phys. 85, 419 (2007).
- QED test : a determination from recoil at LKB-Paris **Looking for PhD student**
- QED numerical calculation…
- QCD lattice calculation…

Proton Size Investigators thank you for your attention



# Prospect : muonic helium spectroscopy at PSI

812 nm
$p_{He}$ = 40 bars

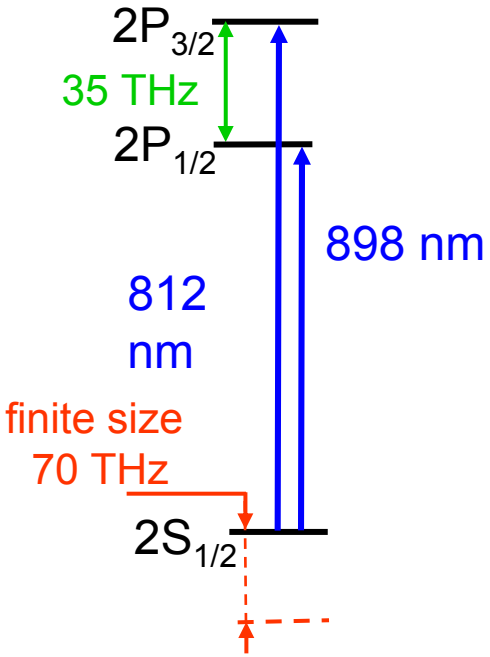



Nuclear Physics A278 (1977) p. 381

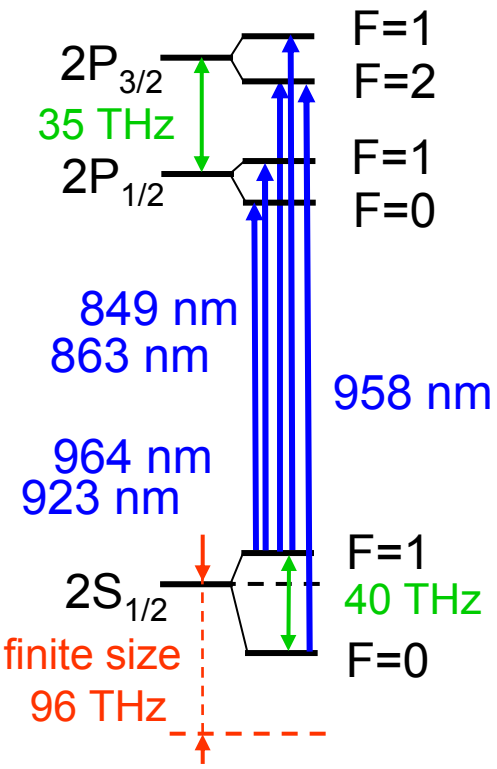
but signal never reproduced
(10 bars, 40 bars)

2011-2013→ muonic helium spectroscopy (4 mbar)

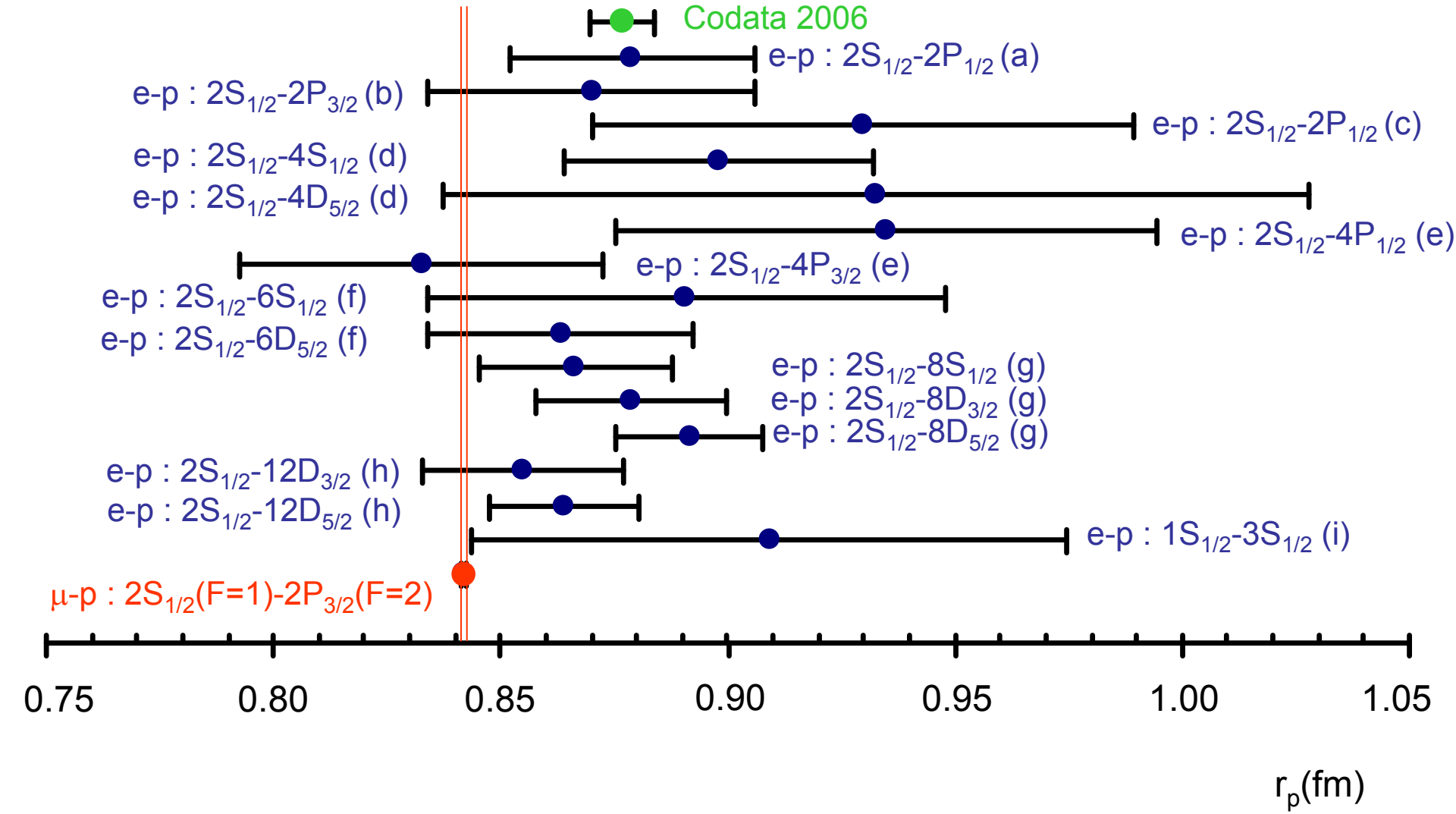
$\mu^4He^+$




$\mu^3He^+$




- $\mu He^+$ spectroscopy + $He^+$ spectroscopy → QED test ($Z\alpha$)
- test of the spectroscopy of muonic system


Codata 2006
e-p : 2S1/2-2P1/2 (a)
e-p : 2S1/2-2P3/2 (b)
e-p : 2S1/2-2P1/2 (c)
e-p : 2S1/2-4S1/2 (d)
e-p : 2S1/2-4D5/2 (d)
e-p : 2S1/2-4P1/2 (e)
e-p : 2S1/2-4P3/2 (e)
e-p : 2S1/2-6S1/2 (f)
e-p : 2S1/2-6D5/2 (f)
e-p : 2S1/2-8S1/2 (g)
e-p : 2S1/2-8D3/2 (g)
e-p : 2S1/2-8D5/2 (g)
e-p : 2S1/2-12D3/2 (h)
e-p : 2S1/2-12D5/2 (h)
e-p : 1S1/2-3S1/2 (i)
μ-p : 2S1/2(F=1)-2P3/2(F=2)
0.75
0.80
0.85
0.90
0.95
1.00
1.05
rp(fm)

# New $r_p$ (μp ($2S_{1/2}(F=1)$ - $2P_{3/2}(F=2)$)) : outcomes

"New physics":

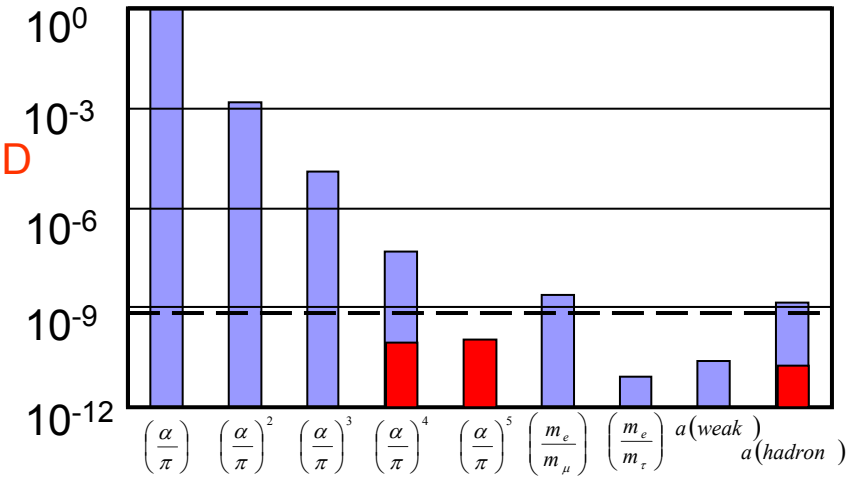
Constrains due to other high precision determination (S.Karshenboim PRL104, 220406 (2010), U.D. Jentschura, Annals of Physics (2010), V. Barger et al, arXiv:10113519V1, …)

*"We consider exotic particles that among leptons, couple preferentially to muons, and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1 and spin-2 particles as an explanation." V.Barger et al (Nov 2010)*

Example of constrain : magnetic moment anomaly of the electron (Gabrielse, Kinoshita)

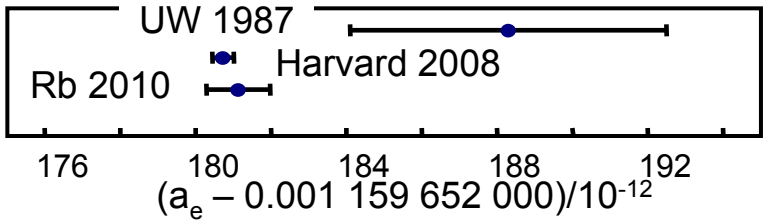
$$a_e = \frac{|\mu_e|}{\mu_B} - 1 = A_1\left(\frac{\alpha}{\pi}\right) + \ldots + A_5\left(\frac{\alpha}{\pi}\right)^5 + a_e(muon) + a_e(tau) + a_e(weak) + a_e(hadron)$$

With our new determination of α : stringent test of QED

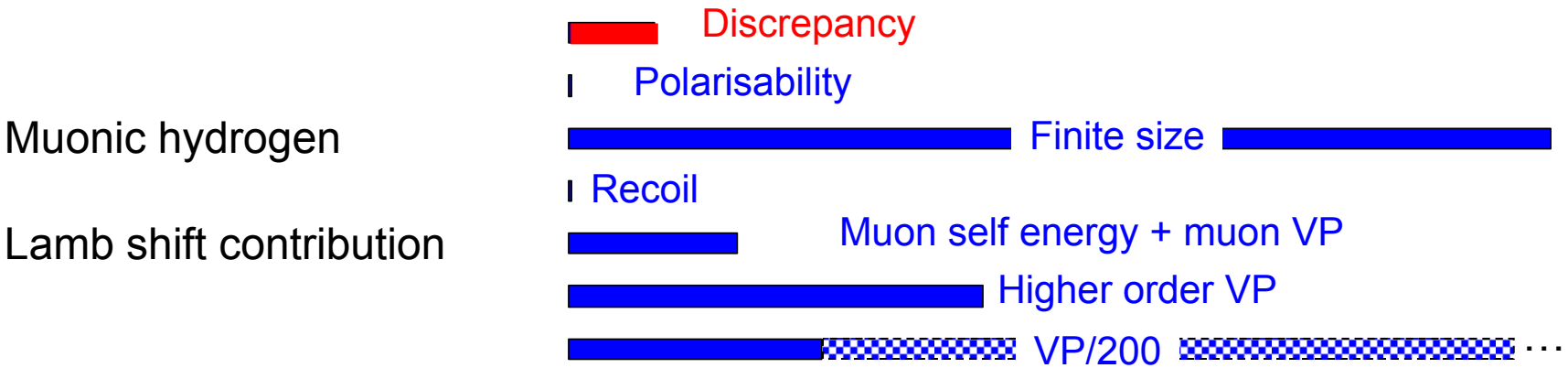



Recoil $v_r = \frac{\hbar k}{m(Rb)}$ $\alpha^2 = \frac{2R_\infty}{c}\frac{m(Rb)}{m_p}\frac{m_p}{m_e}\frac{h}{m(Rb)}$

Paris measurement of α(Rb-2010) ($6.6\times10^{-10}$)
submitted to PRL (15 Dec 2010), arXiv 1012.3627v1



# Lamb shift - form factor – polarizability

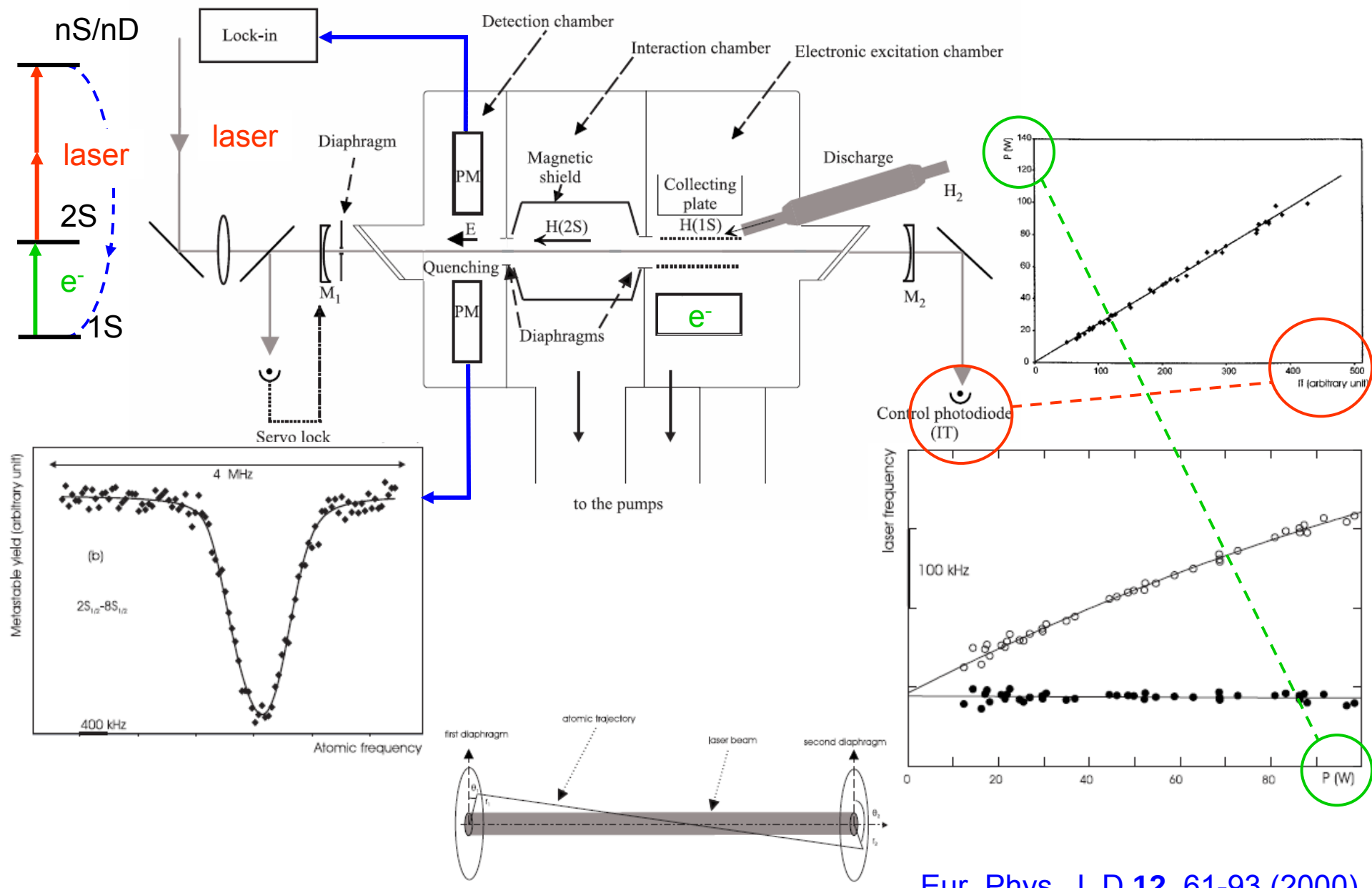



- Dipole form factor $\rho(r) = e^{-\frac{r}{R}}$
- Gaussian form factor $\rho(r) = e^{-\frac{r^2}{R^2}}$

| | polarizability | form-factor |
|---|---|---|
| • 2S state electronic hydrogen (e-p) | 8.7 Hz | |
| • 2S state electronic deuterium (e-d) | 2.7 kHz | |
| • 2S state muonic hydrogen (μ-p) | 3.6 GHz | 500 MHz |
| • 2S state muonic deuterium (μ-d) | 360 GHz | |

A. Antognini thesis, P.Indelicato P.Mohr in preparation

# 2S-nS/nD hydrogen/deuterium spectroscopy



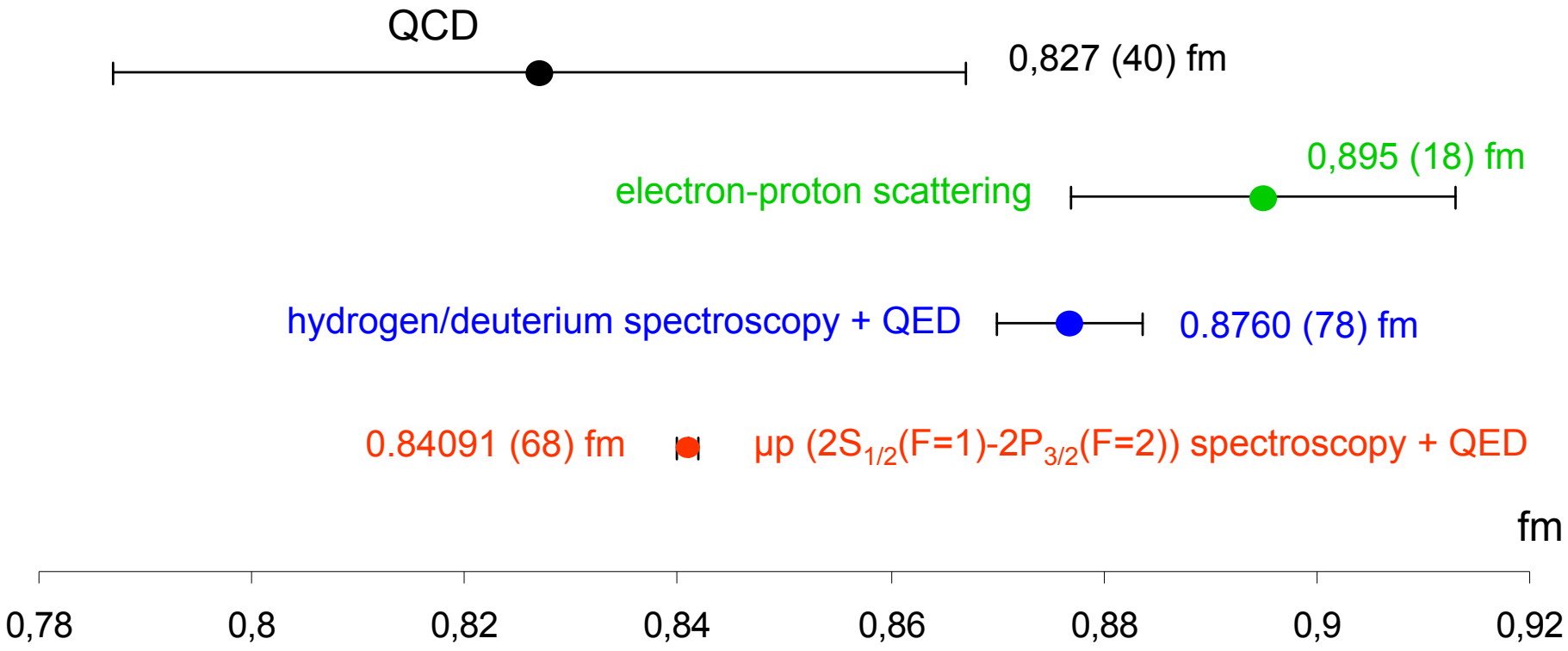

Eur. Phys. J. D **12**, 61-93 (2000)

# Comparison with other best determinations

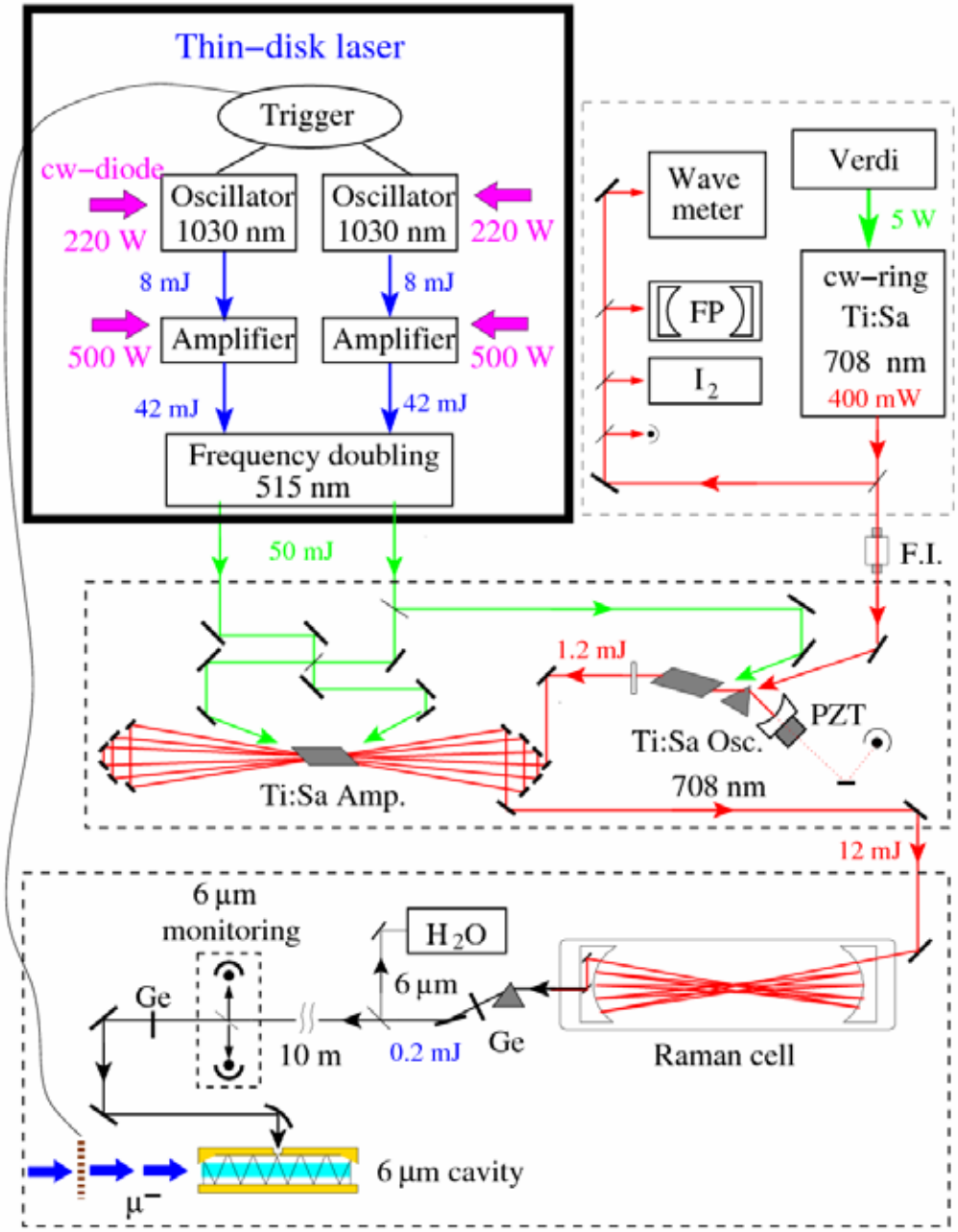
$\nu$ ($\mu$p : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.88 (76) GHz

$\nu_{theo}$= 50772.43(118) – 1263.69 $r_p^2$ + 8.39 $r_p^3$ GHz

$r_p$= 0.84184(36)(56) fm



# Laser chain : principle




- Each single muon triggers the laser system (random trigger)
- 2S lifetime ~1µs → short laser delay
- 6 µm tunable laser pulse (0.2mJ)

$\mu^-$ trigger

↓

Thin disk laser (1030 nm) + LBO (515 nm)

↓

pulsed TiSa oscillator + amplifier
(708 nm determined by cw TiSa seeding)

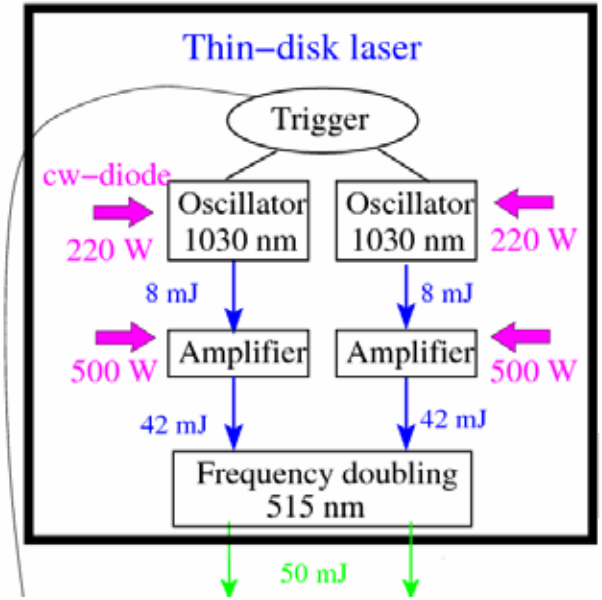
↓

Raman cell (6 µm)

↓

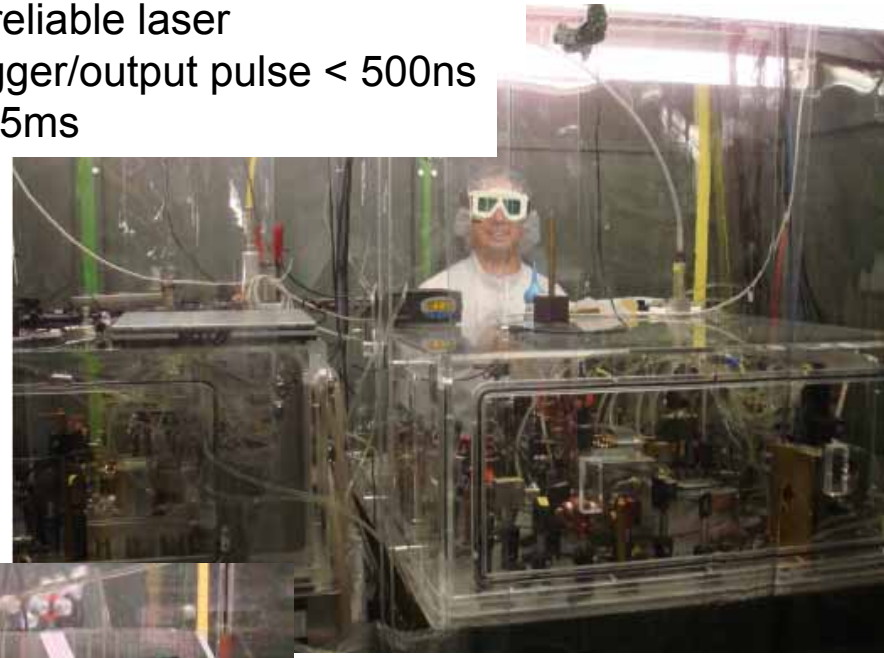
Multipass cavity at 6µm
surrounding the H2 target

# Laser chain : thin-disk Yb:YAG laser

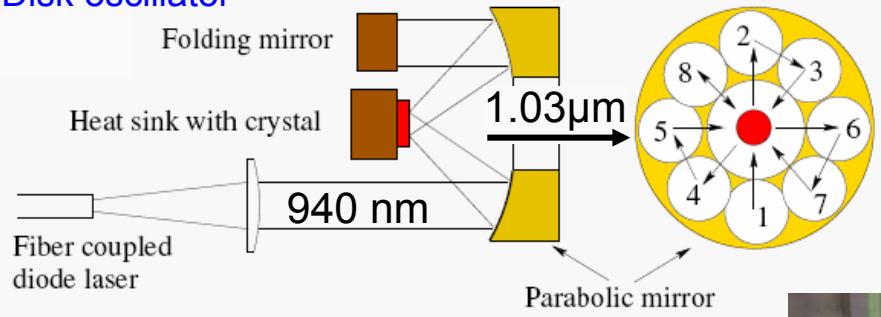


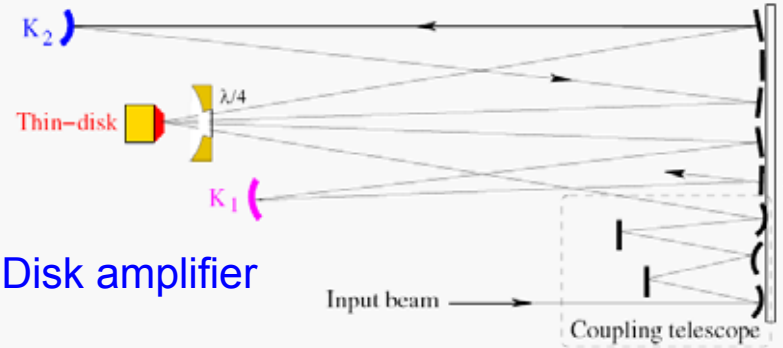

- Q-switch oscillator (cw prelasing mode → short delay between electronic trigger and opt. output (8mJ, xxx ns, delay 250ns)
- 12 pass amplifier (20m propagation) (42mJ)
- LBO frequency doubling 25mJ @ 515nm

- 50 mJ @ 515 nm, reliable laser
- delay electronic trigger/output pulse < 500ns
- random trigger ≥ 1.5ms

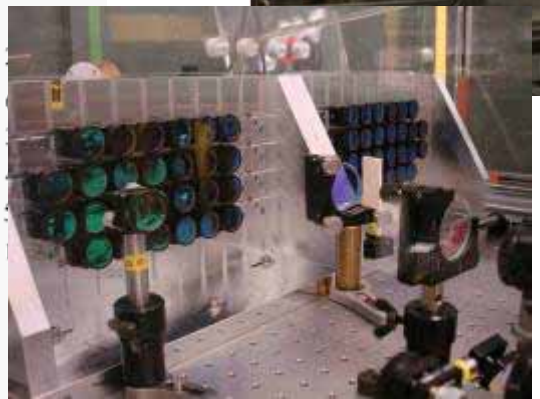


Disk oscillator



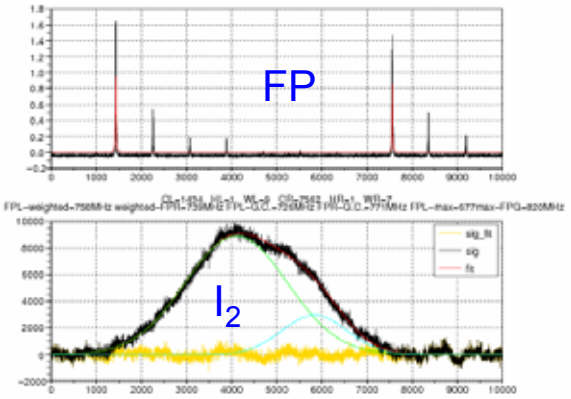

Disk amplifier

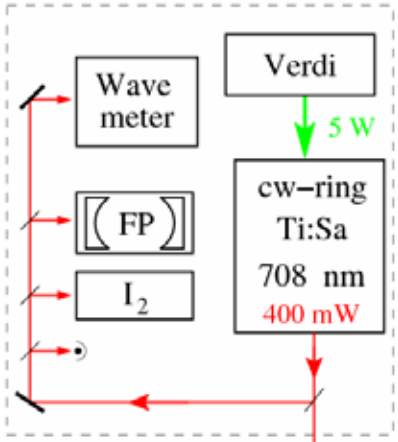


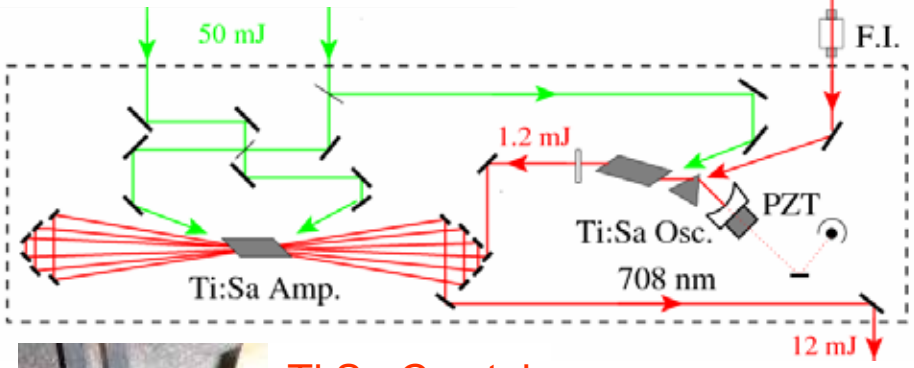
A. Antognini et al,
IEEE J. of Q. Electronics
vol 45, n°8, 2009

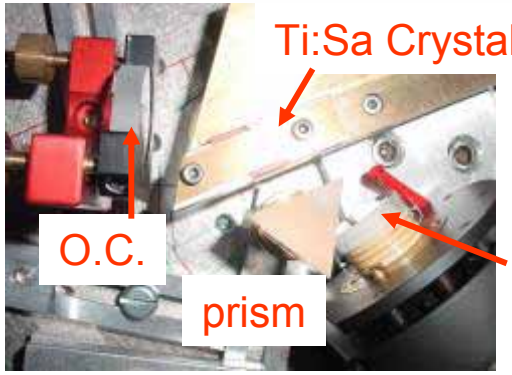
# Laser chain : Ti:Sa

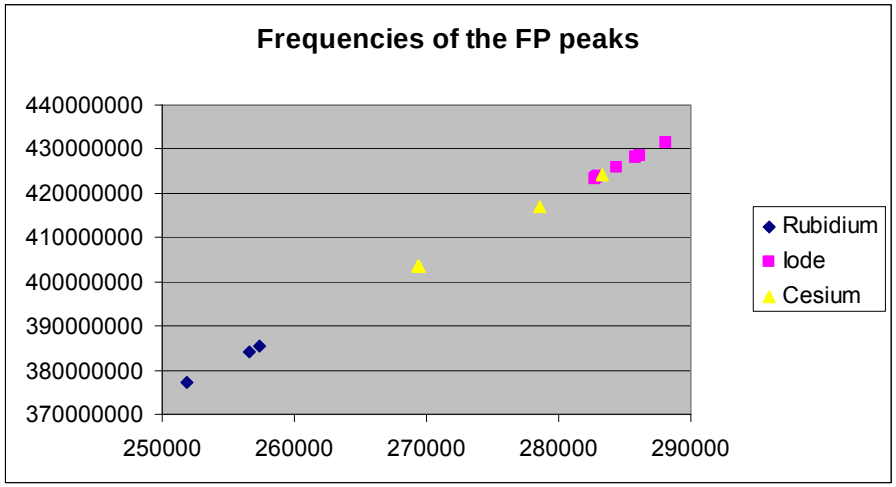
Calibration FP/ $I_2$, Cs, Rb
(700-794nm)



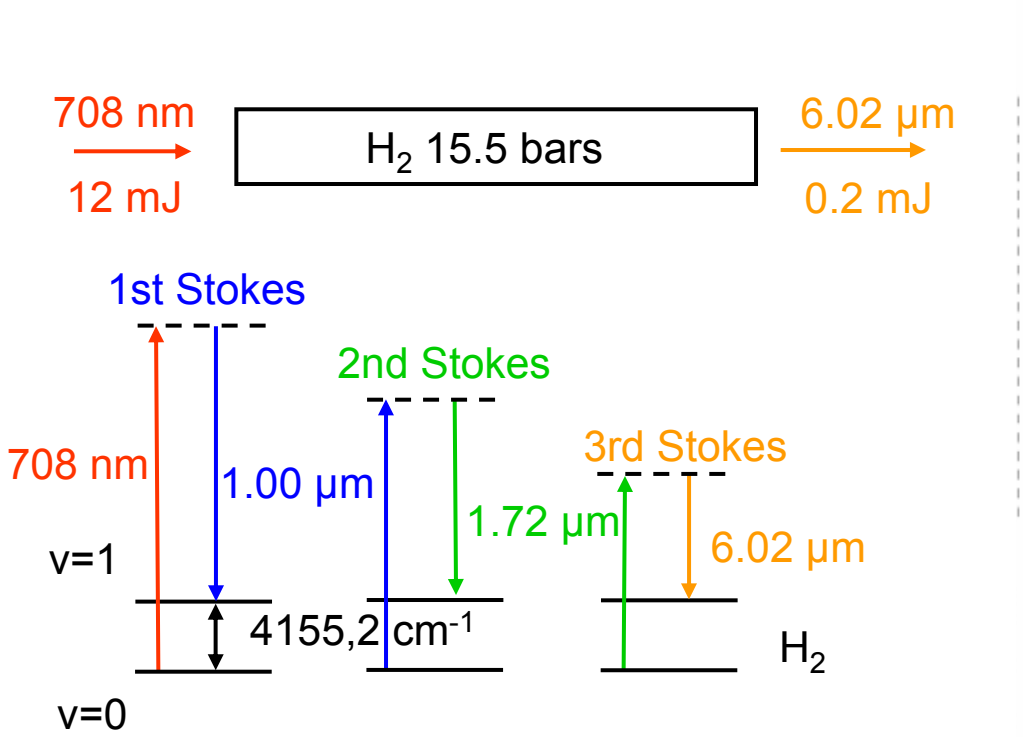



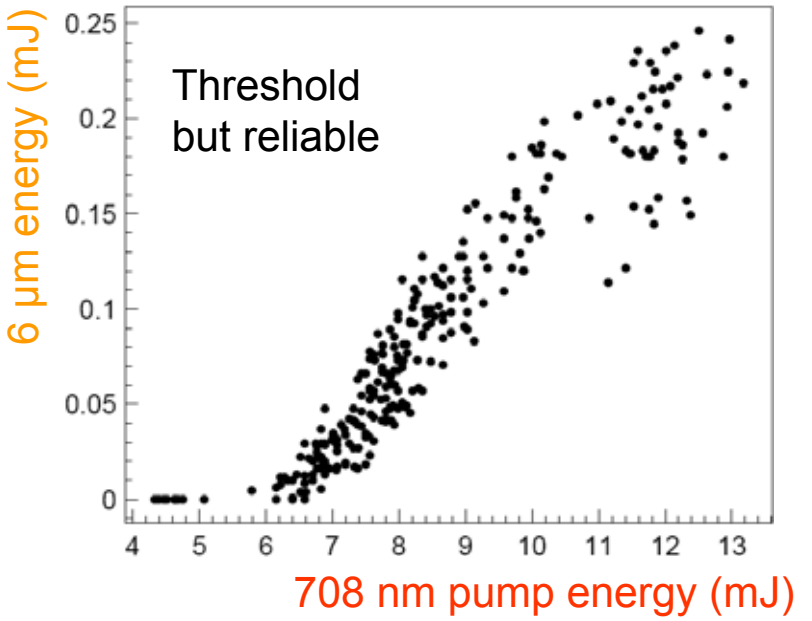



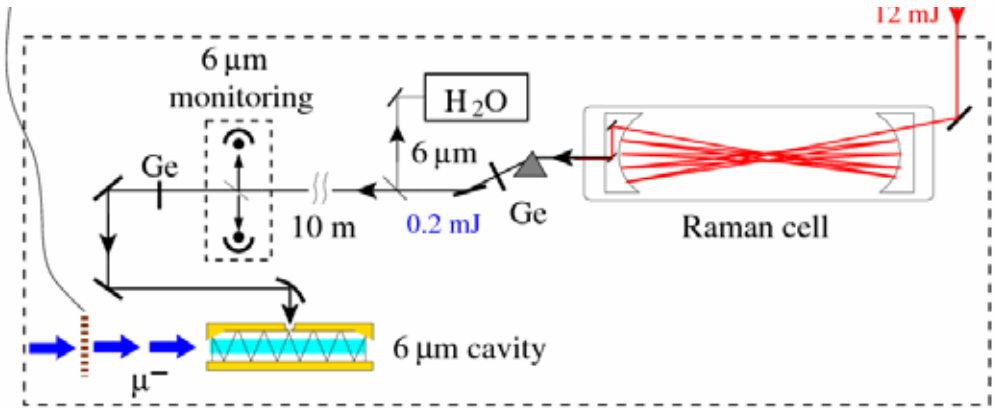

- home made Ti:Sa lasers
- cw-Ti:Sa frequency controlled with FP, atom, molecule (abs. freq. <50MHz)
- short length pulsed oscillator seeded with Cw-Ti:Sa (1.2 mJ, 6 ns, delay 50 ns, $\Delta\nu$ = 200MHz)
- multipass amplifier $\times$ 6 (12 mJ, 5 ns)

Antognini et al, Opt. Comm. 253 (2005) p.362

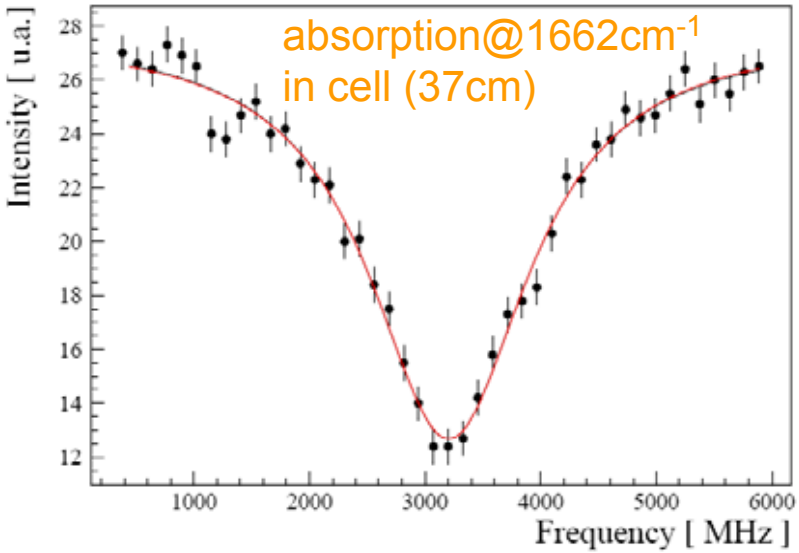
# Laser chain : Raman cell



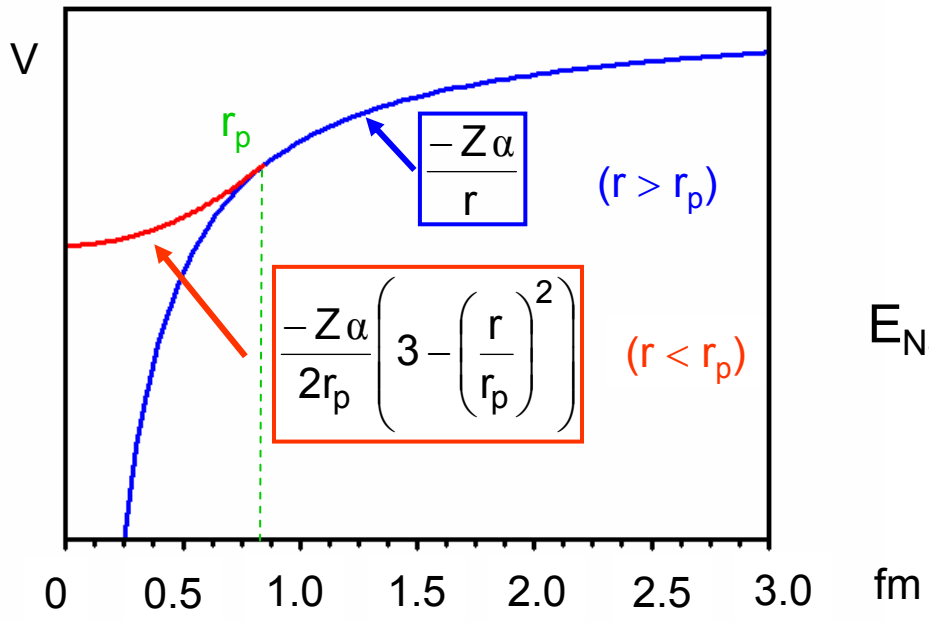




6 µm frequency calibration : $H_20$ lines



- hydrogen spectroscopy (1989, …, 2008)
- muonic hydrogen spectroscopy (2009)




$$E_{NS} = \frac{2}{3}\left(\frac{\mu}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_N}{\lambda\!\!\!^{-}_c}\right)^2 \delta_{l0}$$

# Muonic hydrogen Lamb shift

| Name | Reference | formula | value | | Pachucki [1] | Borie [3] |
|---|---|---|---|---|---|---|
| NR One loop electron VP | | $\frac{8\alpha(Z\alpha)^2}{3\pi n^3} Q_{nl}^{(1)} m_r$ | 205.00740 | | 205.0074 | |
| Relativistic one loop VP | [3] | | | | | 205.0282 |
| NR two-loop electron VP | | $\frac{4\alpha(Z\alpha)^2}{\pi^2 n^3} Q_{nl}^{(2)} m_r$ | 1.50790 | | 1.5079 | 1.5081 |
| NR three-loop electron VP | [12] | $0.08353(1)\frac{\alpha^3(Z\alpha)^2}{\pi^3} m_r$ | 0.00530 | | check | |
| Polarization insertion in two coulomb lines | [4,1] | $0.01244\frac{\alpha^2(Z\alpha)^2}{9\pi^2} m_r$ | 0.15090 | | 0.1509 | 0.1510 |
| Polarisation insertion in two and three coulomb lines | [12] | $0.036506(4)\frac{\alpha^3(Z\alpha)^2}{\pi^3} m_r$ | 0.00230 | | | |
| Relativistic correction | [2,4,1,3] | $\alpha(Z\alpha)^4$ | 0.01690 | | 0.0169 | |
| Wichmann-Kroll | [13,14,3] | $\frac{4\alpha(Z\alpha)^4}{\pi n^3} Q_{nl}^{\rm WK} m_r$ | -0.00103 | | | -0.00103 |
| Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$ | [4,1] | | -0.00500 | 0.0010 | -0.006 | |
| Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$ | | $\left[1.09426\left(-\frac{m}{3m_r}-1\right)-4\times 2.21656\right] \times\frac{\alpha^2(Z\alpha)^4}{\pi^2 n^3}\left(\frac{m_r}{m}\right)^3 m$ | -0.00150 | | | |
| Mixed electron and muon loops | [15] | $\frac{64\alpha^2(Z\alpha)^2}{45\pi^2 n^3}\left(\frac{m_e}{m}\right)^2 Q_{nl}^{(3)} m_r$ | 0.00007 | | | 0.00007 |
| Hadronic polarization $\alpha(Z\alpha)^4 m$ | [16] | $-0.638(22)\frac{4\alpha(Z\alpha)^4}{15\pi^2 n^3}\left(\frac{m_r}{m}\right)^3 m\delta_{l0}$ | 0.01080 | 0.0004 | 0.0113 | 0.0110 |
| Hadronic polarization $\alpha(Z\alpha)^5 m$ | | | 0.00005 | | | |
| Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m$ | | | -0.00002 | | | |
| Recoil contribution | [17] | $\frac{(Z\alpha)^4 m_r^3}{2n^3 M^2}\left(\frac{1}{j+\frac{1}{2}}-\frac{1}{l+\frac{1}{2}}\right)(1-\delta_{l0})$ | 0.05750 | | 0.0575 | 0.0575 |
| Radiative corrections of order $\alpha^n(Z\alpha)^k m$ | [11] | Tables 3.2, 3.3, 3.7, 3.8, 3.9 | -0.66770 | | -0.6677 | -0.66788 |
| Recoil corrections of order $\alpha(Z\alpha)^n\frac{m}{M}m$ | [11] | Table 4.1 | -0.04470 | | -0.045 | -0.04497 |
| Radiative Recoil corrections of order $\alpha(Z\alpha)^n\frac{m}{M}m$ | [11] | Table 5.1 | -0.00960 | | -0.0099 | -0.0096 |
| Nuclear structure correction of order $(Z\alpha)^5$ | [1] | $-\frac{0.095(18)}{n^3}\delta_{l0}$ meV | 0.01200 | | 0.012 | 0.015 |
| Light by light electron loop contribution | | | 0.00135 | 0.00135 | | |
| Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m$ | | $\frac{-0.00152}{n^3}\delta_{l0}$ meV | 0.00019 | | | |
| Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m$ | | $\frac{0.000092}{n^3}\delta_{l0}$ meV | -0.00001 | | | |
| Sum | | | 206.04310 | 0.00275 | 206.0353 | 206.04739 |
| terms dependent on the nuclear radius from Ref. [11] (contributions in meV | | | | | | |
| Leading nuclear size contribution | | $\frac{2(Z\alpha)^4}{3n^3} m_r^3 < r^2 > \delta_{l0}$ | -5.1975 | $< r_p^2 >$ | | |
| Nuclear size correction of order $(Z\alpha)^5$ | [4,1] | $-\frac{(Z\alpha)^5}{3n^3} m_r^4 < r^3 >_{(2)} \delta_{l0}$ | 0.0347 | $< r_p^3 >$ | $0.0363 < r_p^3 >$ | $0.0347 < r_p^3 >$ |
| Radiative corrections to nuclear finite size effect | [1] | $\alpha(Z\alpha)^4 m_r^3 < r_p^2 >$ | -0.0275 | $< r_p^2 >$ | | $-0.0273 < r_p^2 >$ |
| Nuclear size correction of order $(Z\alpha)^6 < r_p^2 >$ | [1] | $-\langle \ln\frac{Z\alpha}{\eta}\rangle \frac{2(Z\alpha)^6}{3n^3} m_r^3 < r_p^2 >$ | | $< r_p^2 >$ | | |
| Nuclear size correction of order $(Z\alpha)^6 < r_p^4 >$ | [1] | $\frac{2}{3}\frac{2(Z\alpha)^6}{3n^3} m_r^5 (< r_p^2 >)^2$ | | $< r_p^2 >^2$ | | |
| Total $< r^2 >$ contribution | | | -5.225 | | | |

Table 7. All known contributions from Eides et al.[11])

Form factor (dipole, Gaussian, …)

d:4, G:1

P. Indelicato et al.: Summary of QED calculations for muonic hydrogen and deuterium

# Hydrogen spectroscopy

Transition energy : $\left(E_n - E_p\right) = \left(E_n^{Dirac} - E_p^{Dirac}\right) + \left(E_n^{Recoil} - E_p^{recoil}\right) + \left(L_n - L_p\right)$

in which : $E_{Dirac}$ and $E_{Recoil}$ have an exact formulation prop. to $R_\infty$

$L_n$=F(QED, $R_p$) Lamb shift

✓ Q.E.D. = series expansion of $\alpha$, $Z\alpha$, $\ln(Z\alpha)$ which depends on n as $1/n^3$

$$\Delta E = \left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^4}{n^3} m_e c^2 \left[B_{40} + (Z\alpha)B_{50} + (Z\alpha)^2\left(L^3 B_{63} + L^2 B_{62} + LB_{61} + B_{60}\right) + \ldots\right]$$

$$\approx \quad \ldots \quad \left[0{,}54 - 0{,}16 + 5\times 10^{-5}\left(-280 - 50 + 500 - 100\right) + \ldots\right]$$

$$avec \quad L = \ln\left[(Z\alpha)^{-2}\right]$$

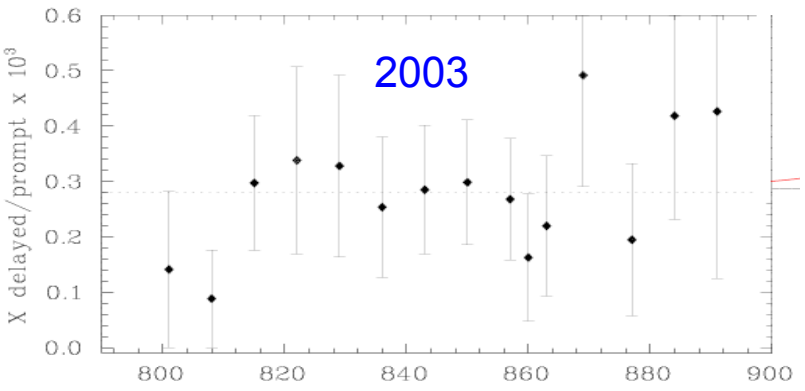
✓ Distribution charge radius contributions vary as $1/n^3$

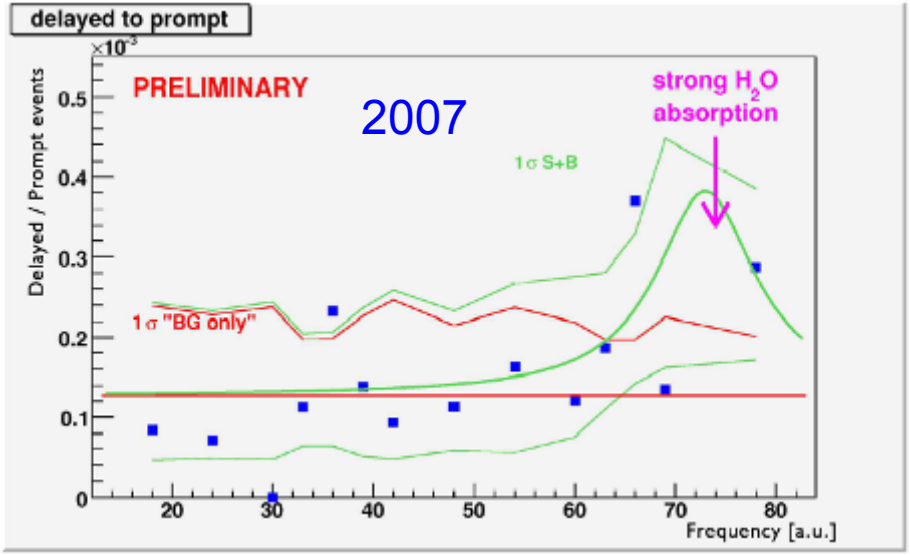
$$E_{NS} = \frac{2}{3}\left(\frac{\mu}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_p}{\lambda\!\!\!^{-}_c}\right)^2 \delta_{l0}$$

- Q.E.D. calculations are right ! : 2 experimental data $\rightarrow R_\infty$, $R_p$
- are Q.E.D. calculations right ? : 3 experimental data $\rightarrow$ $R_\infty$, $R_p$, Q.E.D.

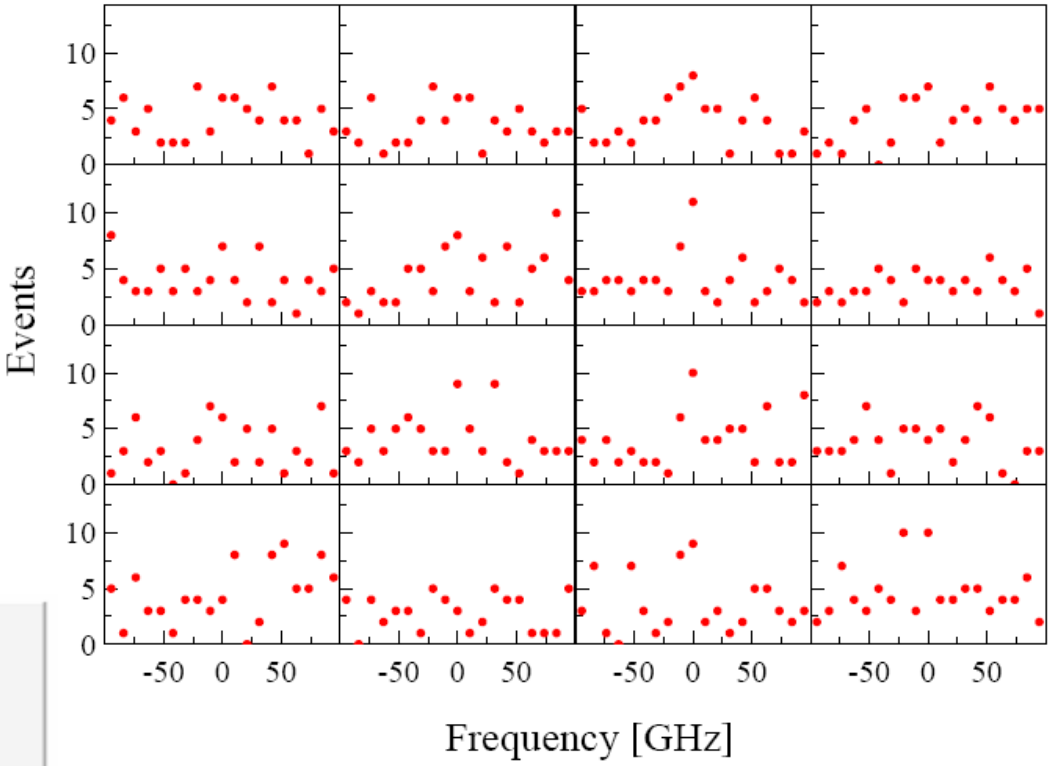
# Resonance searches (2002, 2003, 2007)

2002 : Run with all the components of the experiment at the same time (search for resonance 10 hours).



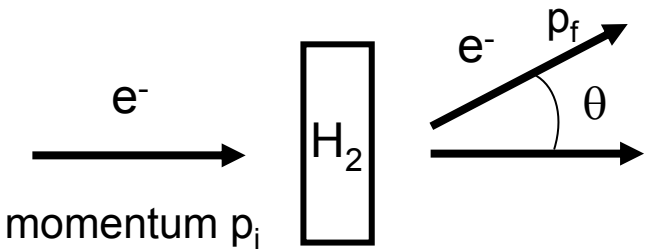

Simulation 2003 :




sig. 0.5 c/hour
noise 0.5 c/hour

from A. Antognini Phd thesis

# electron-proton scattering




$$\vec{q} = \vec{p}_f - \vec{p}_i \qquad q = 2\,p\,\sin\left(\frac{\theta}{2}\right)$$

$$\frac{d\sigma}{d\Omega}(E_i,\theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i,\theta)\right]}_{\text{Rutherford}} G(q^2)$$

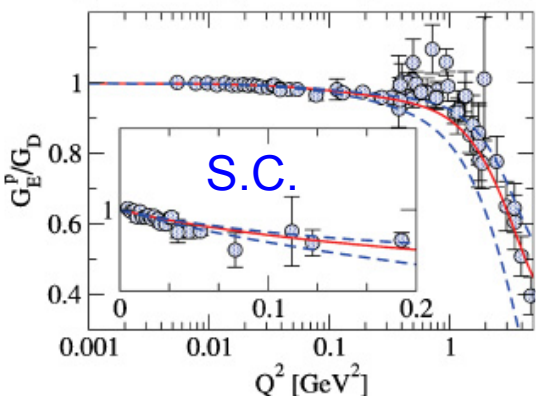
$$G(q^2) = \int d^3r\, e^{iqr}\,\frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6}q^2$$

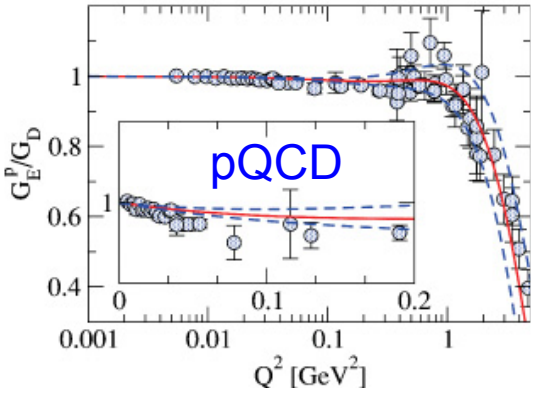
Sick 2003 :

$$G_e(q) = \cfrac{1}{1+\cfrac{q^2 b_1}{1+\cfrac{q^2 b_2}{1+\ldots}}}$$

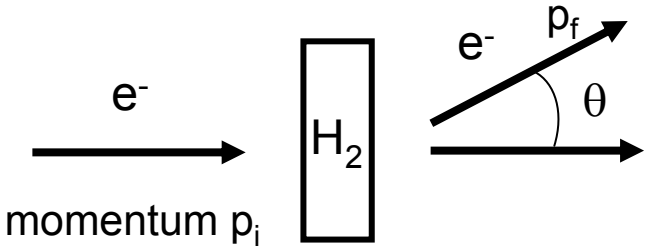
Belushkin et al 2007 :

$$G_D(Q^2) = \frac{1}{\left(1+\frac{Q^2}{m_D^2}\right)^2}$$



# electron-proton scattering

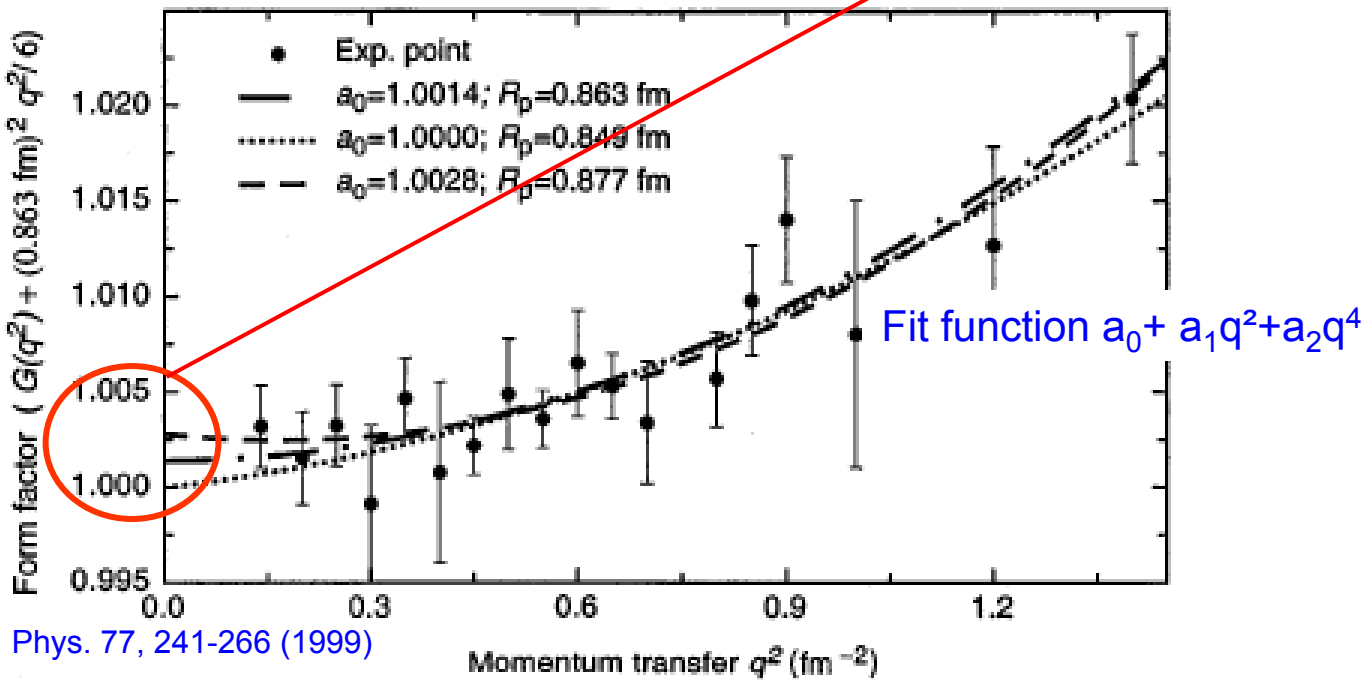



momentum $p_i$
energy $E_i$

$$\vec{q} = \vec{p}_f - \vec{p}_i \qquad q = 2p\sin(\frac{\theta}{2})$$

$$\frac{d\sigma}{d\Omega}(E_i,\theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i,\theta)\right]}_{\text{Rutherford}} G(q^2)$$

$$G(q^2) = \int d^3r\, e^{iqr}\frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6}q^2$$



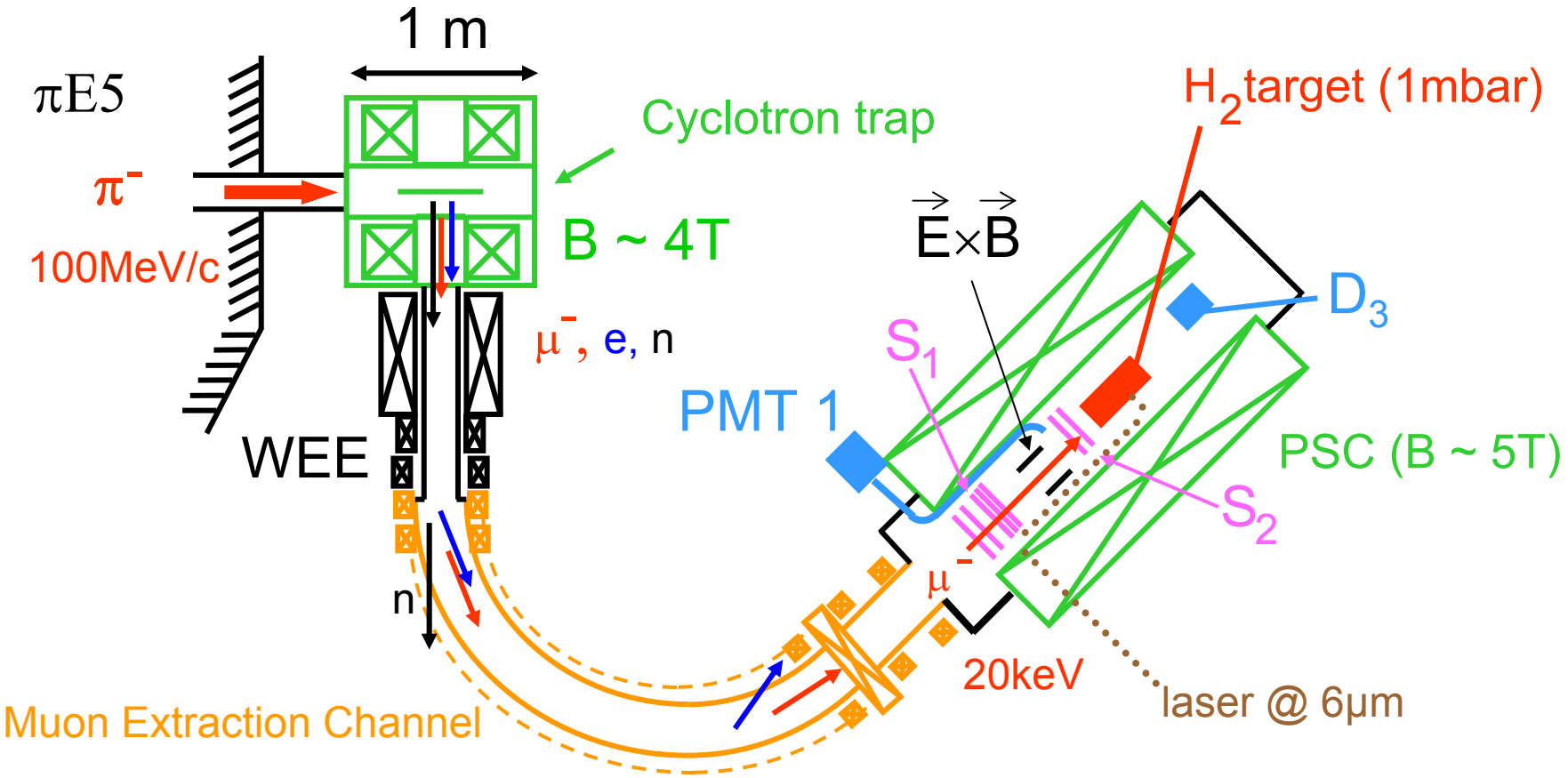

Fit function $a_0 + a_1q^2 + a_2q^4$

see S. Karshenboim in Can. J. Phys. 77, 241-266 (1999) and refs therein

# muonic hydrogen source

PAUL SCHERRER INSTITUT

p (500Mev, 2mA) → carbon target → $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ (300 $\mu^-$/s)



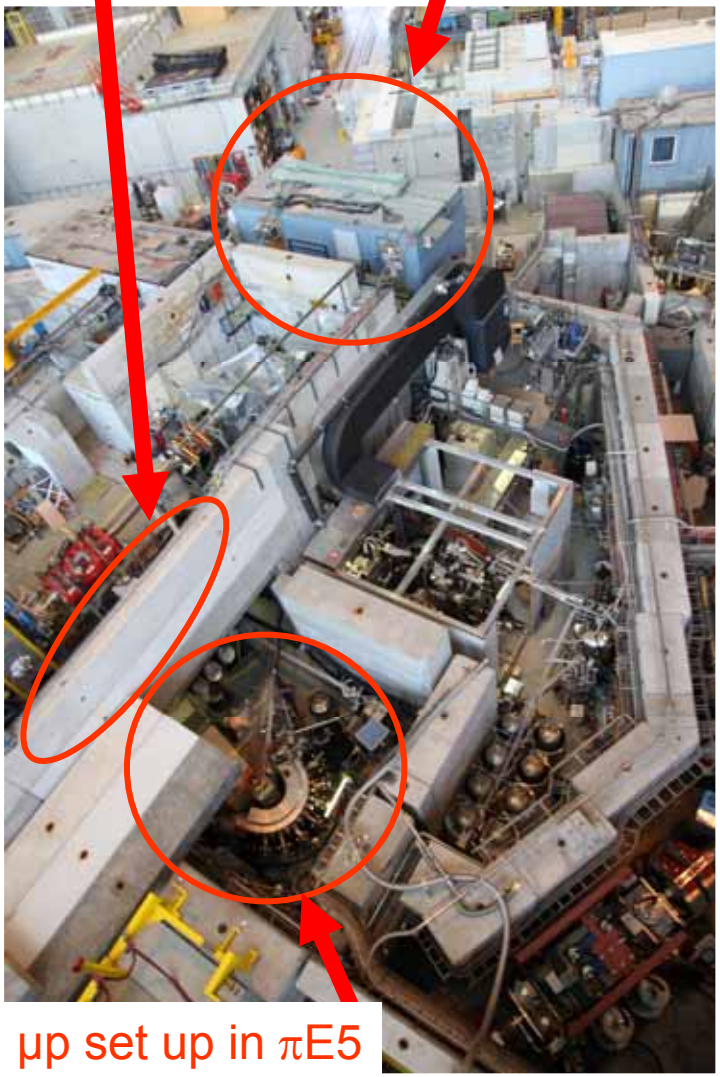

$S_1$ $S_2$ → T.O.F. → laser trigger

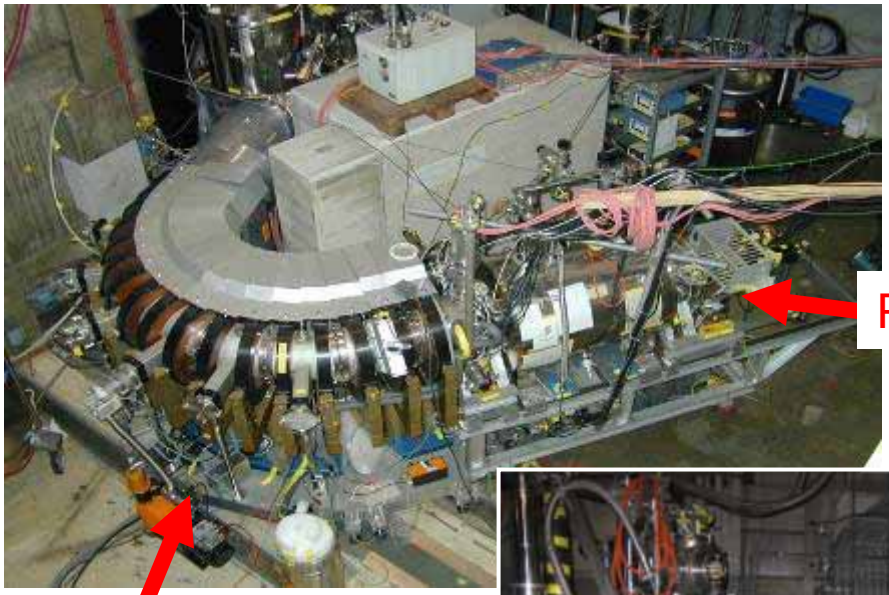
# muon beam apparatus

$\pi^- \rightarrow \mu^- + \nu_\mu$

laser hut below concrete blocks

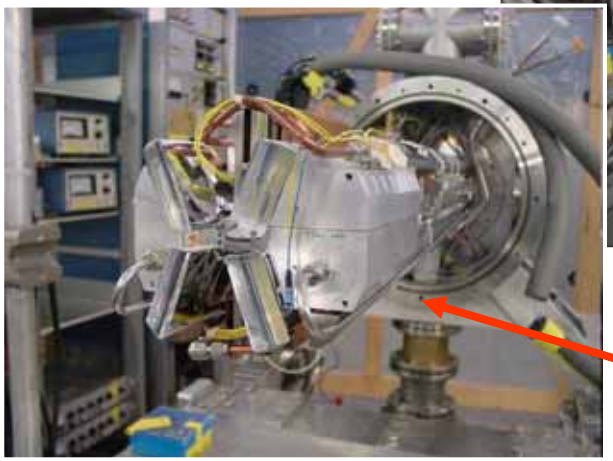
counting room





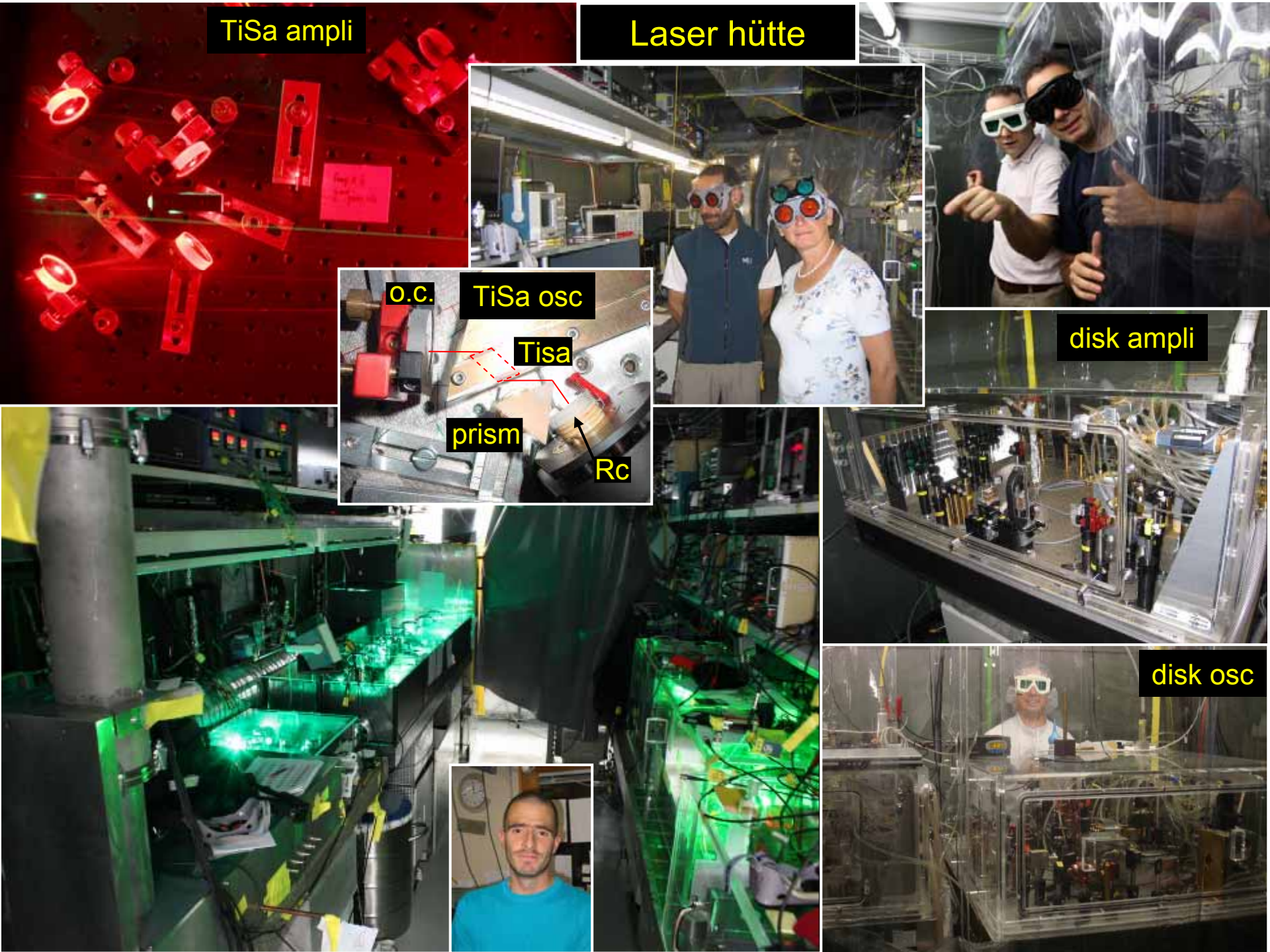
PSC solenoid,

Muon extraction channel



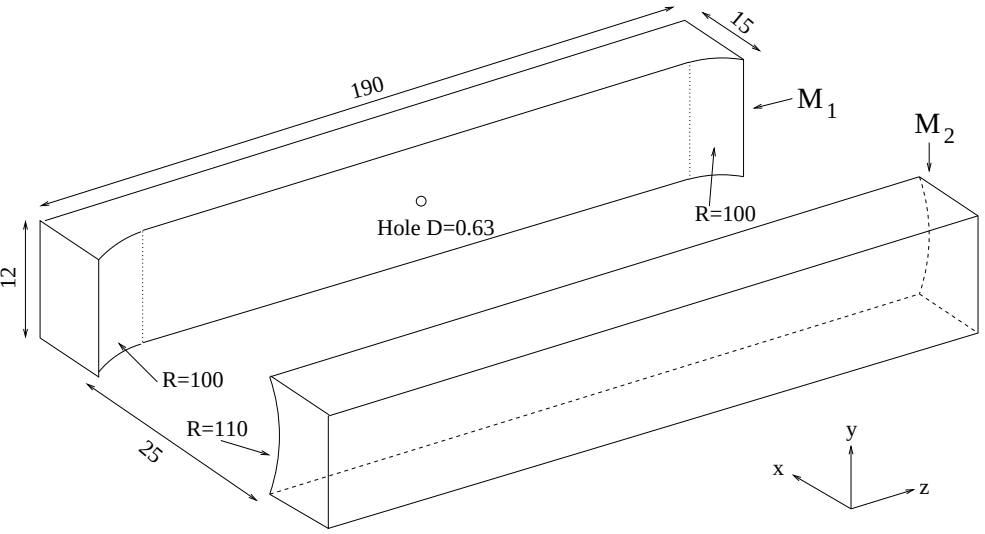
H2 target, laser cavity, detectors

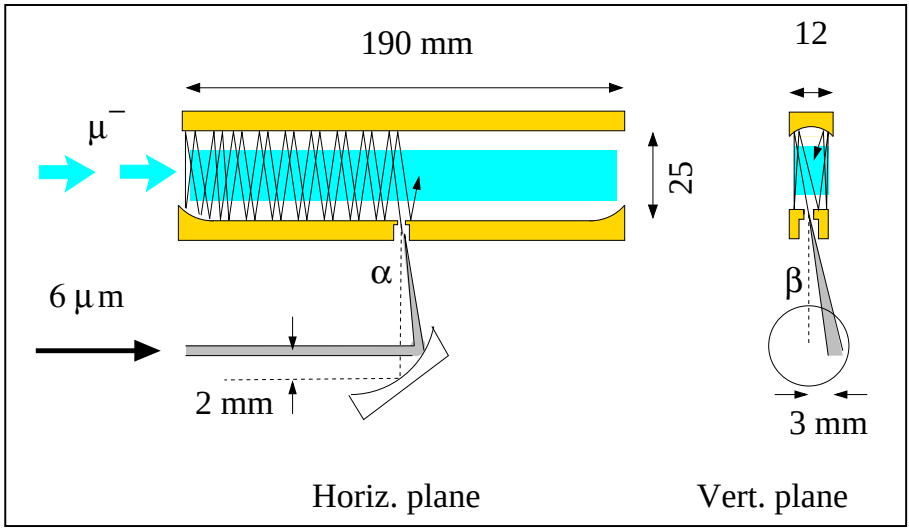
μp set up in πE5


TiSa ampli
Laser hütte
o.c.
TiSa osc
Tisa
prism
Rc
disk ampli
disk osc

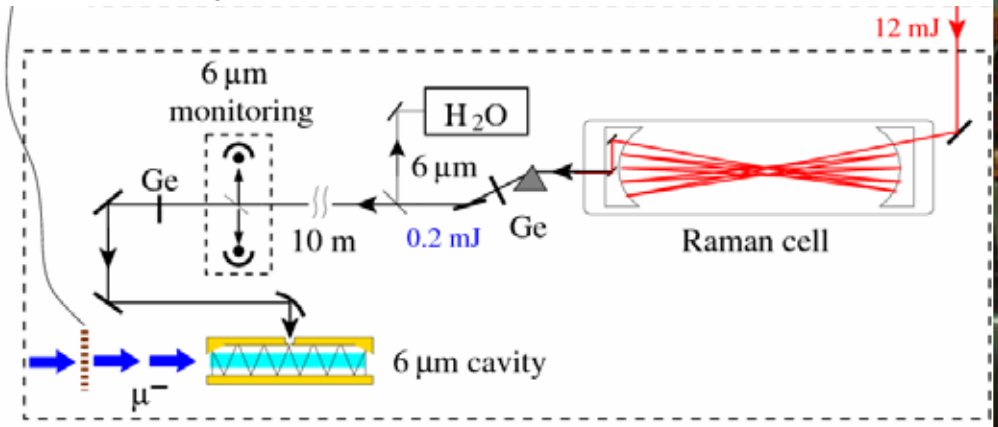
# Laser chain : multipass cavity

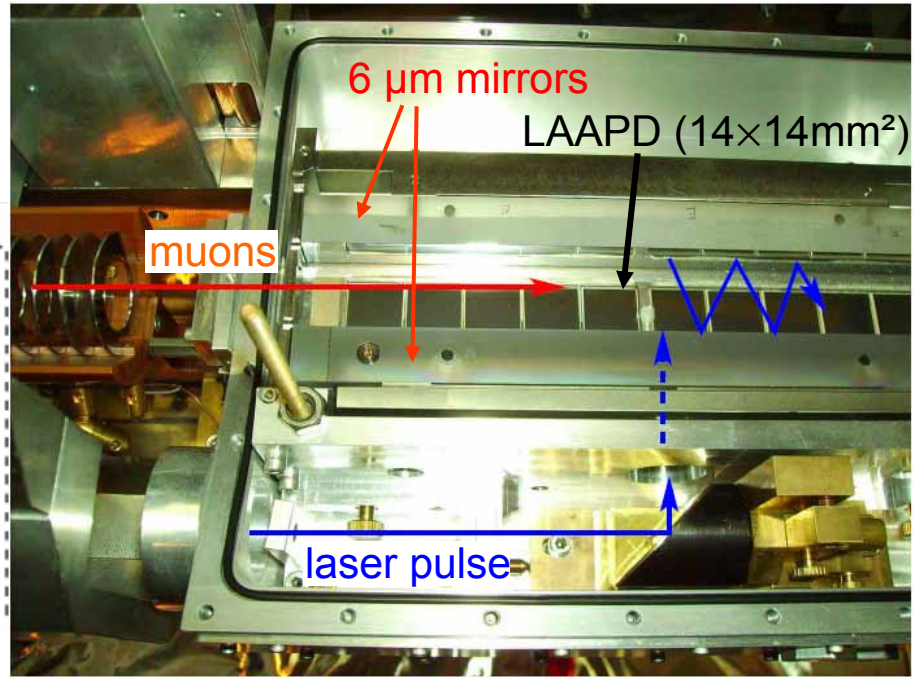
→ illuminate at 6 µm all the muon stopping volume (5×15×190 mm$^3$)




- coupling through a 0.63mm diameter hole
- R=99.90% at 6 µm
- 1000 reflections
- 0.15mJ injected → 2S-2P saturated



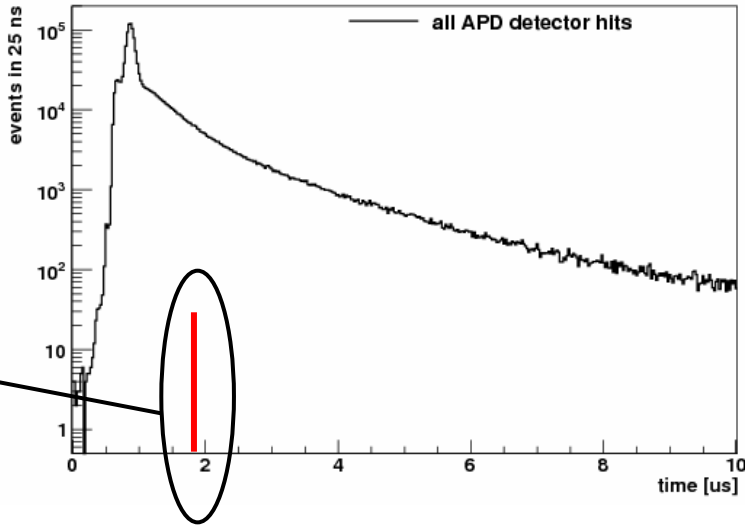



# X-rays analysis → event gate sorting → noise rejection

Example : FP 900 - 11 hrs meas.

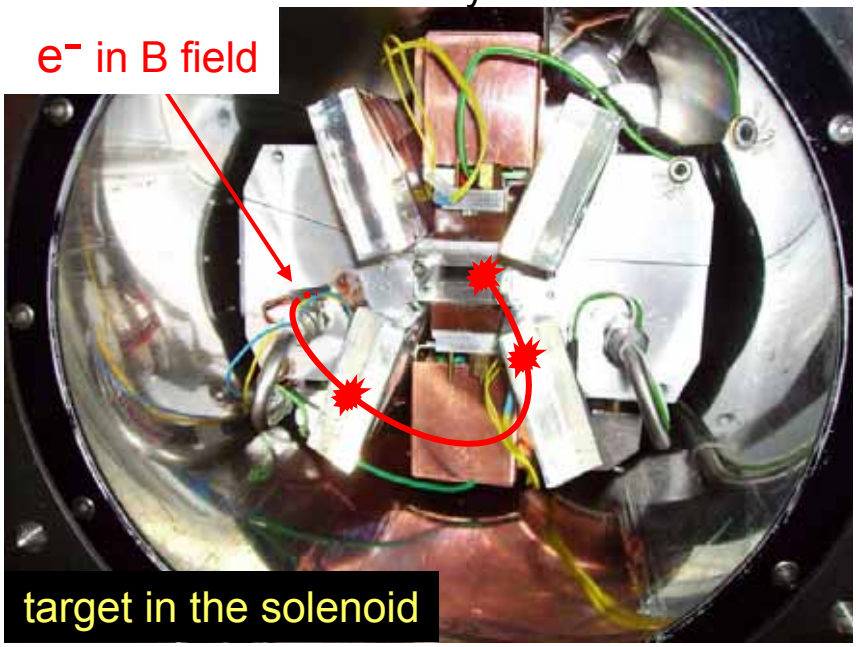
1.56 million detector events

expected 2-3 laser induced events/hour !



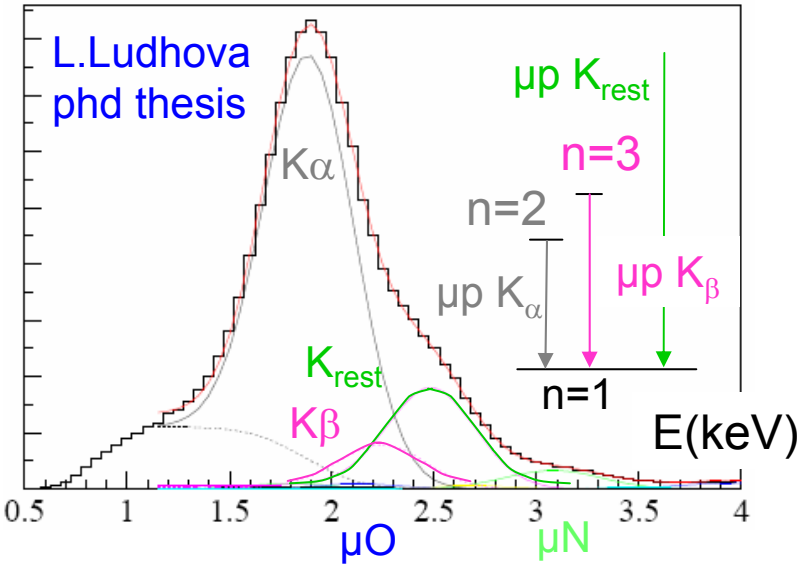

## time signature in LAAPD

- photon < 10keV → 1 shot in the LAAPD
- $e^-$ in B = 5T → many counts in detectors




## energy signature in LAAPD

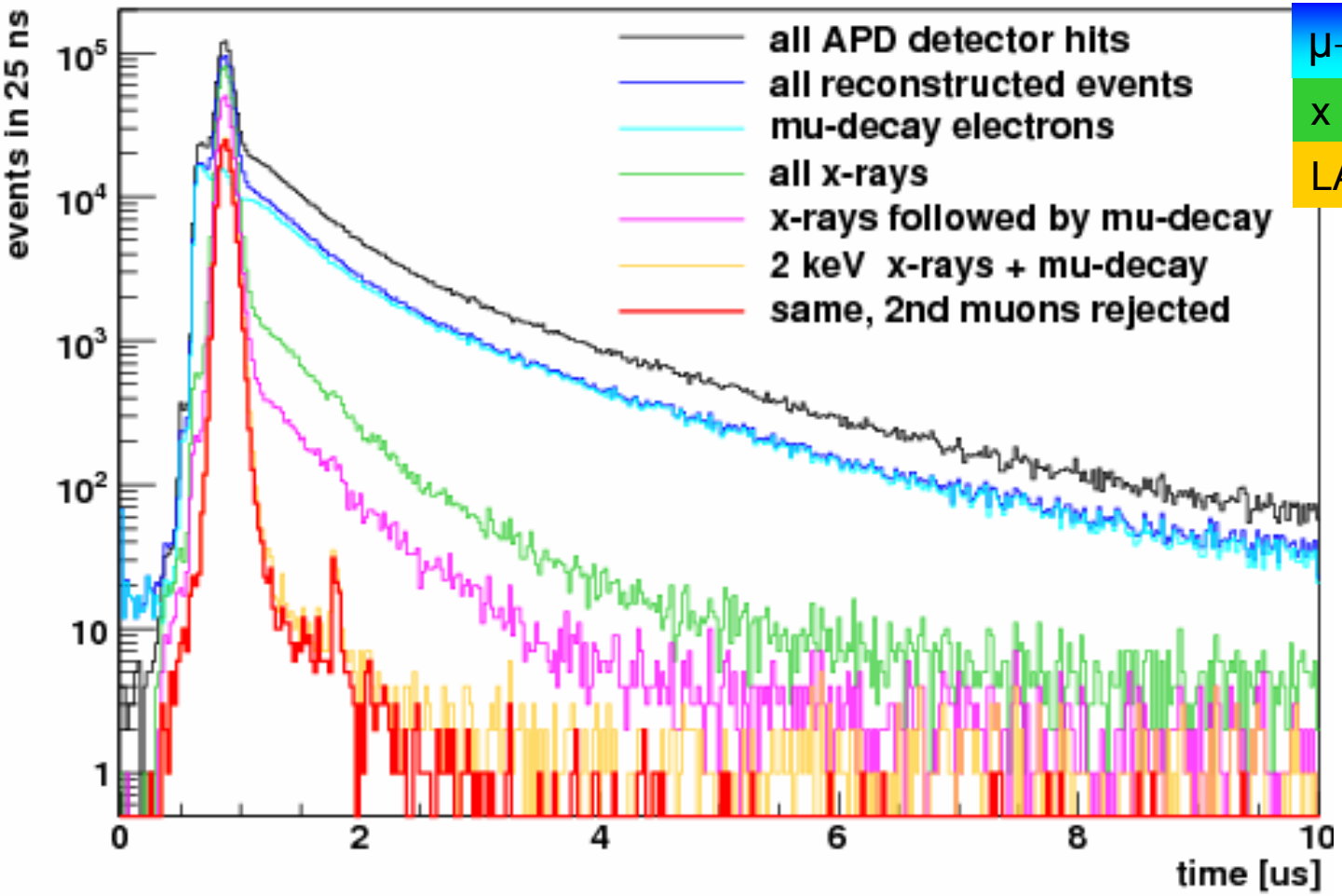
- E > 8keV ⇔ electron
- 1keV < E < 8keV ⇔ X ray
- E<1keV ⇔ neutron



# X-rays analysis → noise rejection

Example : FP 900 - 11 hrs meas.

- 400 $\mu^-$/s
- 240 laser shot/s
- 860 000 laser shot/hour
- 1.56 million detector clicks
- 19600 clicks in the laser region
- expected 2-3 laser induced events/hour !




$\mu \rightarrow e\ \nu_\mu\ \bar{\nu}_e$

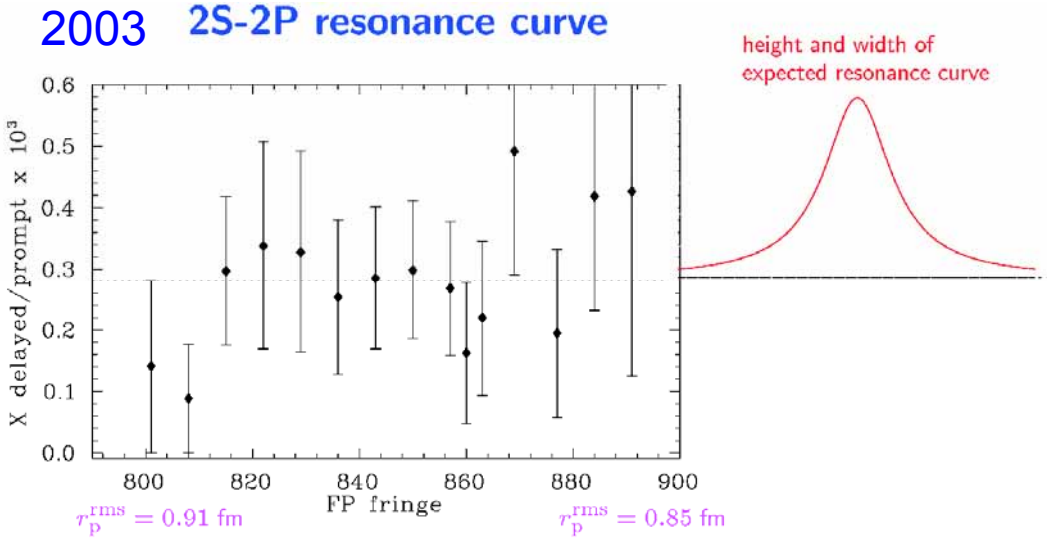
x rays multiplicity 1

LAAPD energy resolution

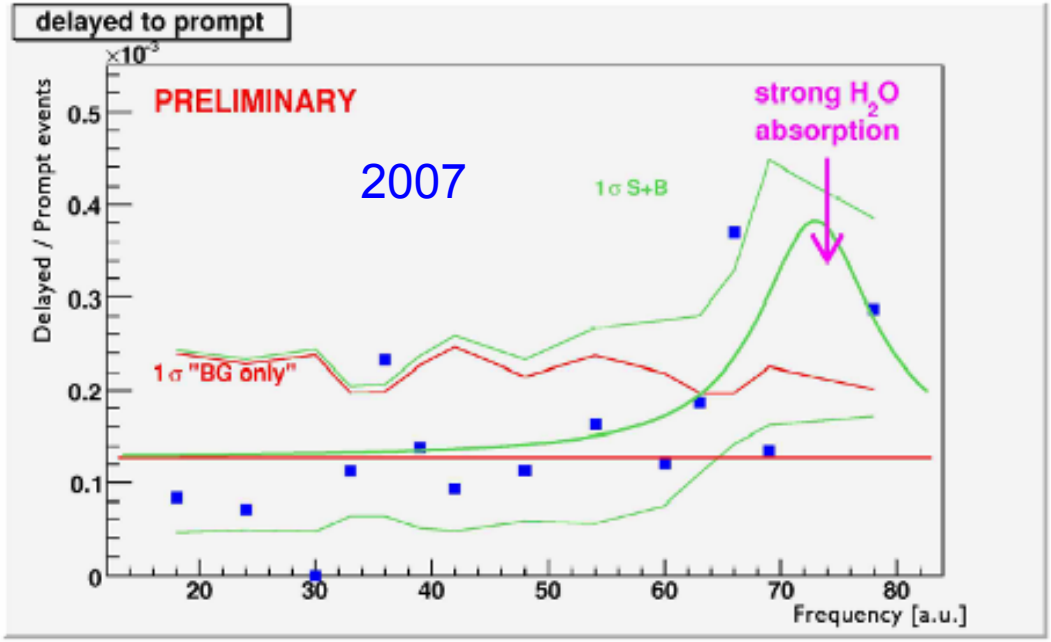
# Resonance searches (2002, 2003, 2007)

2002 : Run with all the parts at the same time (search for resonance 10 hrs).

2003 **2S-2P resonance curve**

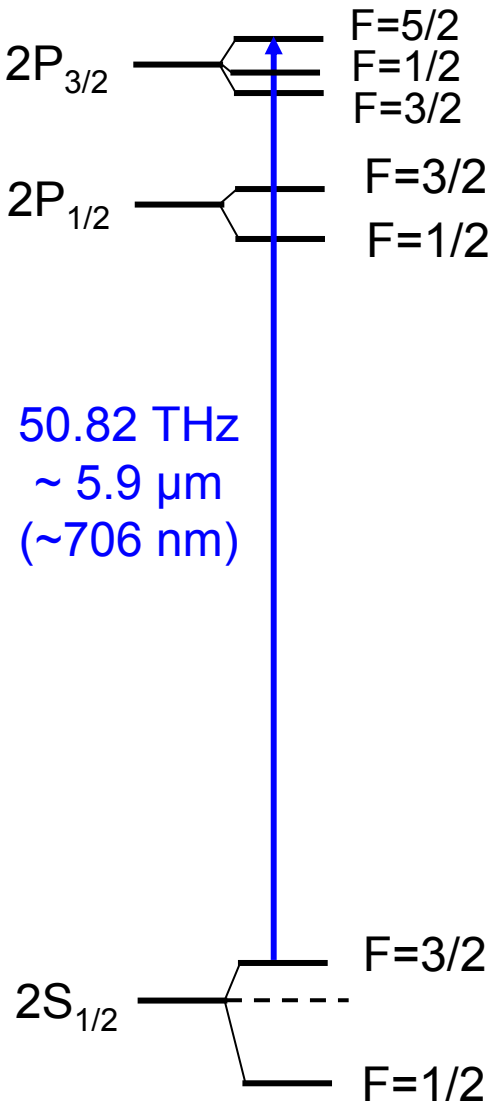



2 “new” XeCl lasers

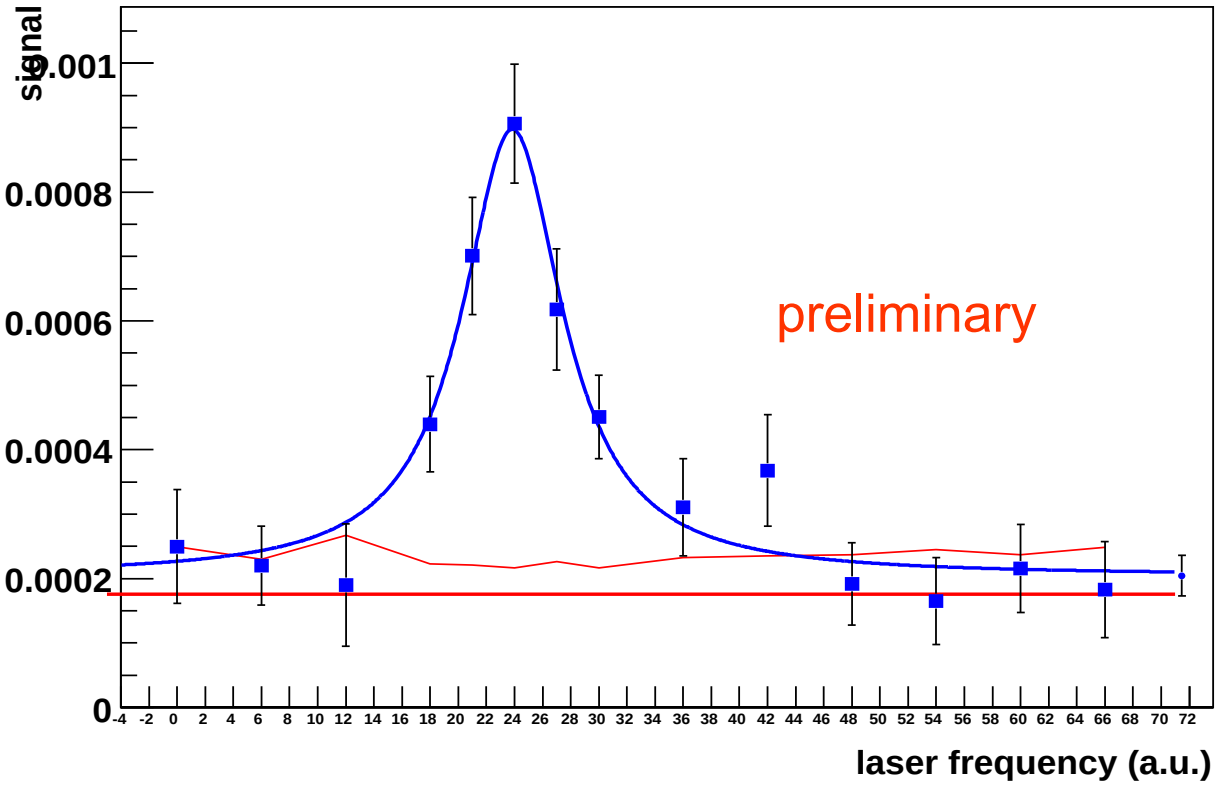



new disk laser

# muonic deuterium : $2S_{1/2}(F=3/2)$- $2P_{3/2}(F=5/2)$

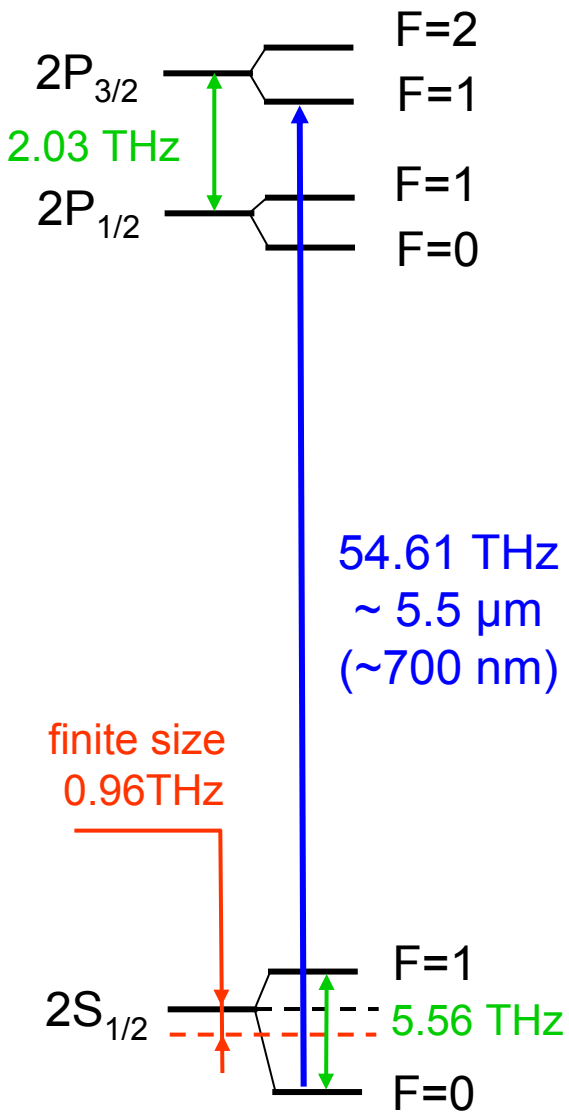



- on line signal, preliminary
- frequency position again off

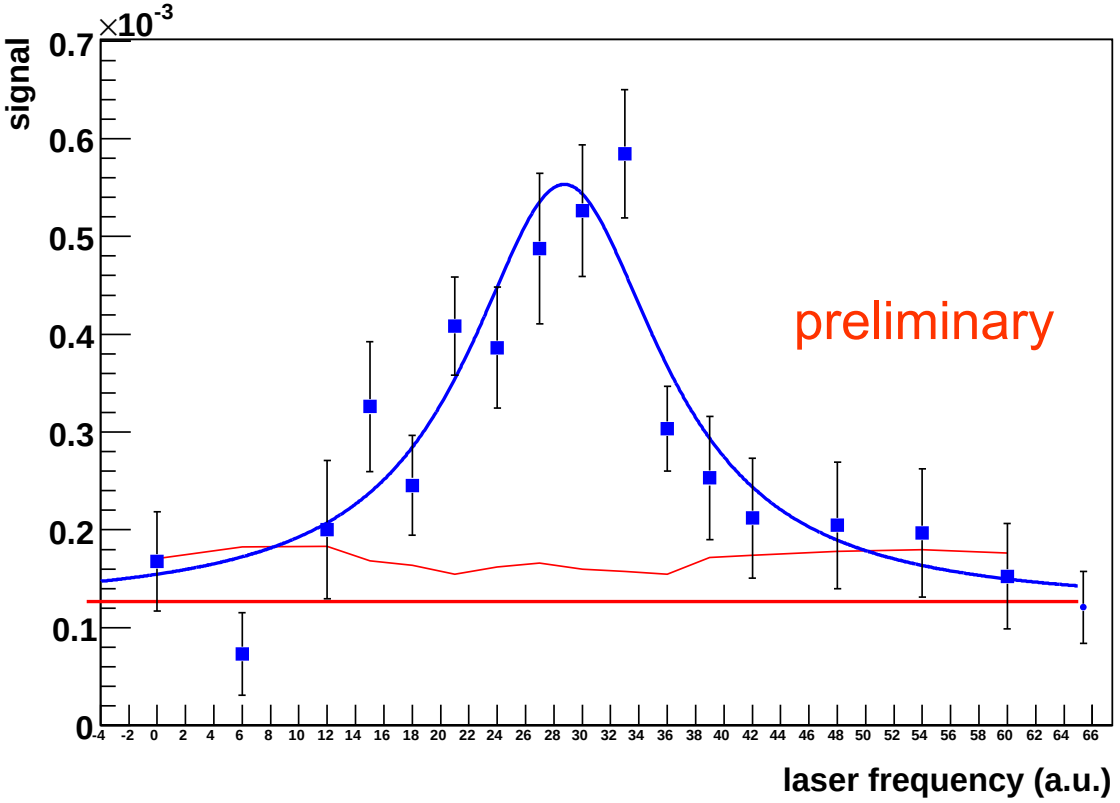



Not at the position estimated with new $r_p$ and isotopic shift → deuteron polarizability

# muonic hydrogen: $2S_{1/2}(F=0)$- $2P_{3/2}(F=1)$

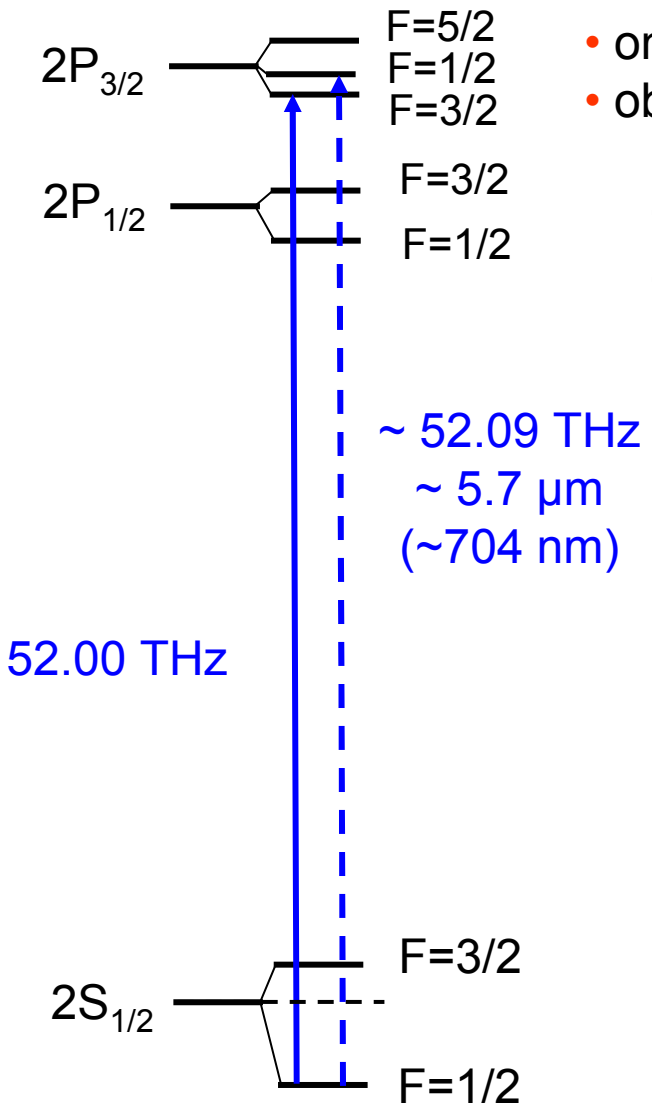



- measured position fits with our proton radius (preliminary)
- laser worked even better at 5.5 µm

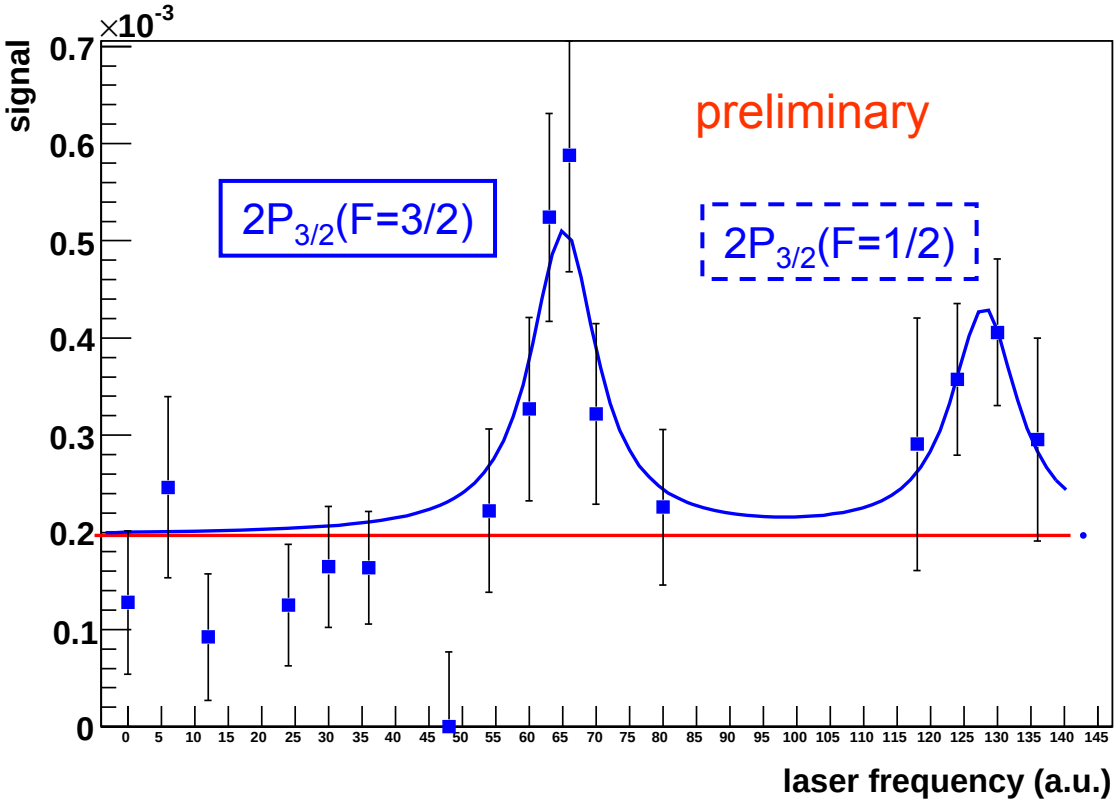



~ at the position deduced with new $r_p$ → hfs : Zeemach radius (few %)

# muonic deuterium : $2S_{1/2}(F=1/2)$- $2P_{3/2}(F=3/2)$ and $2S_{1/2}(F=1/2)$- $2P_{3/2}(F=1/2)$



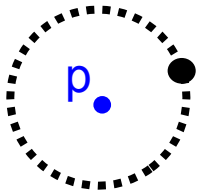

- on line signal, preliminary
- observation of 2 more lines in the last 2 days of beam line




"observation" → check calculation in μd

# Muonic hydrogen spectroscopy

Exotic atom




$\frac{m_\mu}{m_e} = 207$

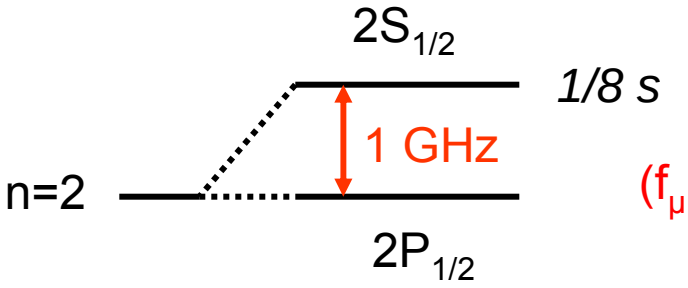
radius : ~ $a_0/200$

$$E_{NS} = \frac{4}{3}\left(\frac{m_r}{m_e}\right)^3 \frac{R_\infty hc}{n^3}\left(\frac{r_p}{a_0}\right)^2 \delta_{l0}$$

Lamb shift = self-energy + vacuum polarization + proton radius

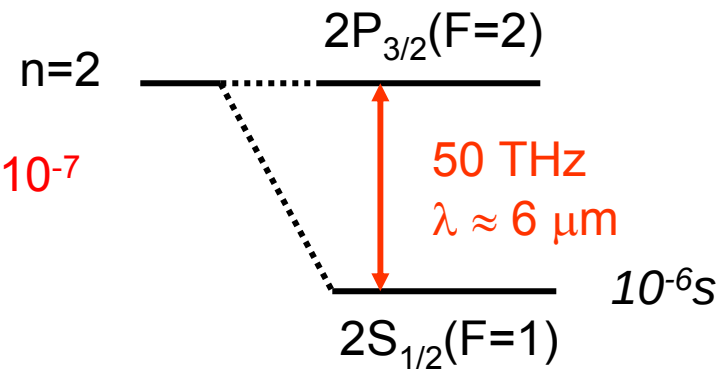
| 2S-2P | self-energy | vacuum pol. | $r_p$ | total |
|---|---|---|---|---|
| e-p | 1084.1 MHz | -26.9 MHz | 0.146 MHz | 1057.8 MHz |
| μ-p | 0.17 THz | -50.94 THz | 0.96 THz | -49,81THz |

electronic hydrogen : e-p




$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \approx 10^{-7}$

muonic hydrogen : μ-p

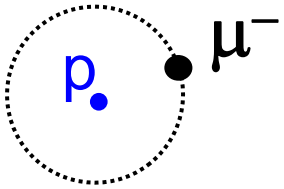



$\Delta E_{2S-2P} = 50.7724(12) - 1.2637\ r_p^2 - 0.0084\ r_p^3 = 49.8083\ (149)\text{THz}$

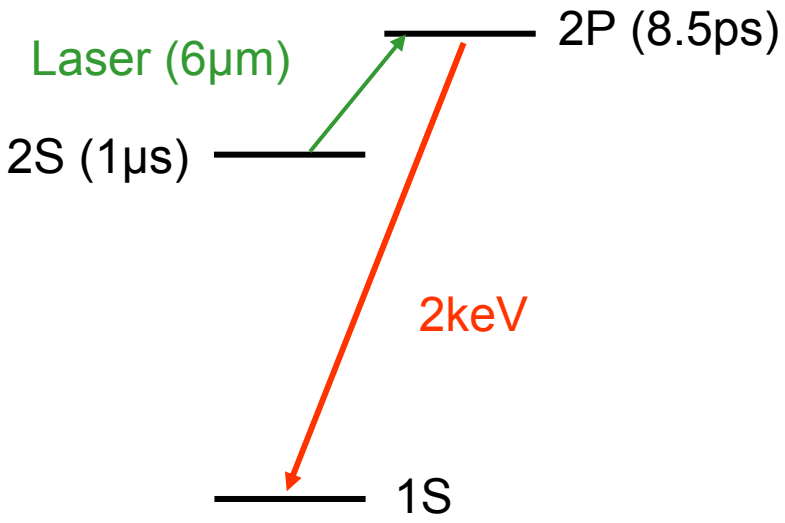
with $r_p$=0.8760(68)fm

# muonic hydrogen 2S Lamb shift determination of the “proton radius”

Exotic atom



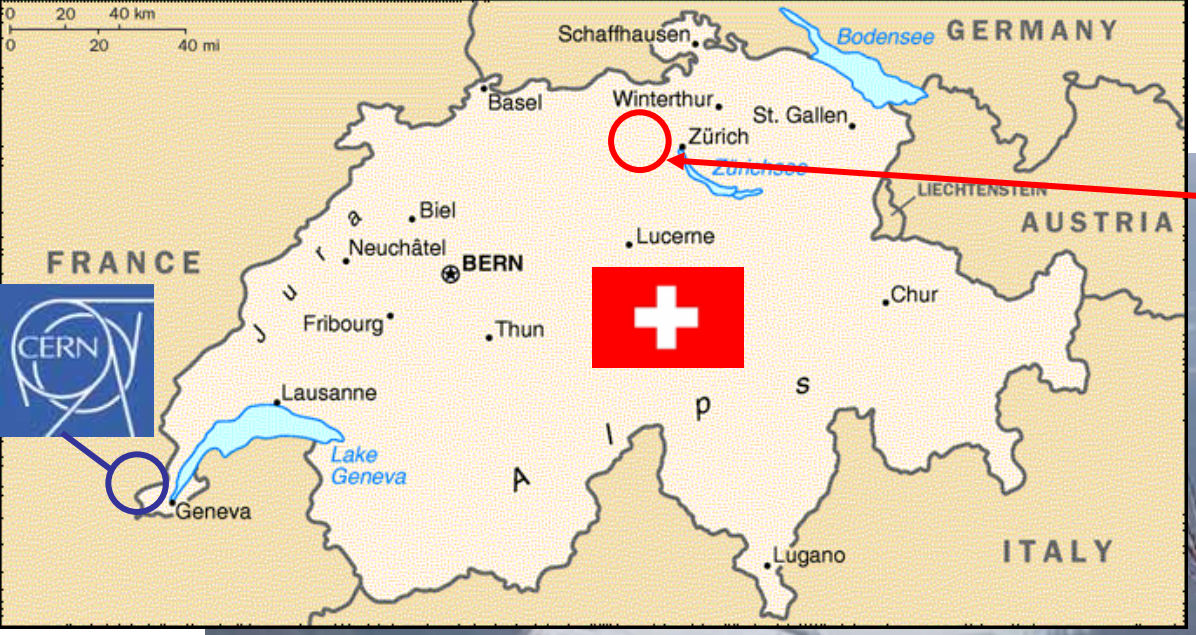

Experiment




Challenges

- production of muonic hydrogen in 2S
- powerful triggerable 6μm laser
- small signal analysis

Aim : better determination of proton radius $r_p$




PAUL SCHERRER INSTITUT

PSI

Experimental hall

# Outline

## 1. Introduction

- proton properties
- experimental proton size determinations
- muonic hydrogen

## 2. Experiment

- muonic hydrogen source
- laser system
- analysis

## 3. Results

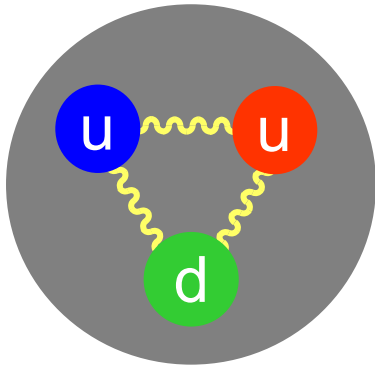
- signal
- proton radius from muonic hydrogen spectroscopy

## 4. Future

# Proton

## Structure

2 quarks up (2/3 e) + 1 quark down (-1/3 e) + strong interaction (gluons)




## Proton charge radius : $r_p$

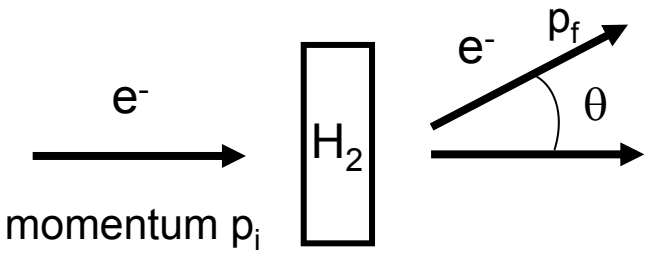
$$r_p = \sqrt{\langle r^2 \rangle} \qquad \langle r^2 \rangle = \frac{\int r^2 \rho(r)\, d^3r}{\int \rho(r)\, d^3r}$$

$\rho(r)$ : proton charge distribution

## QCD Calculations

- proton mass (< 4%)
- $r_p$ (< 5%)

## Electron-proton scattering



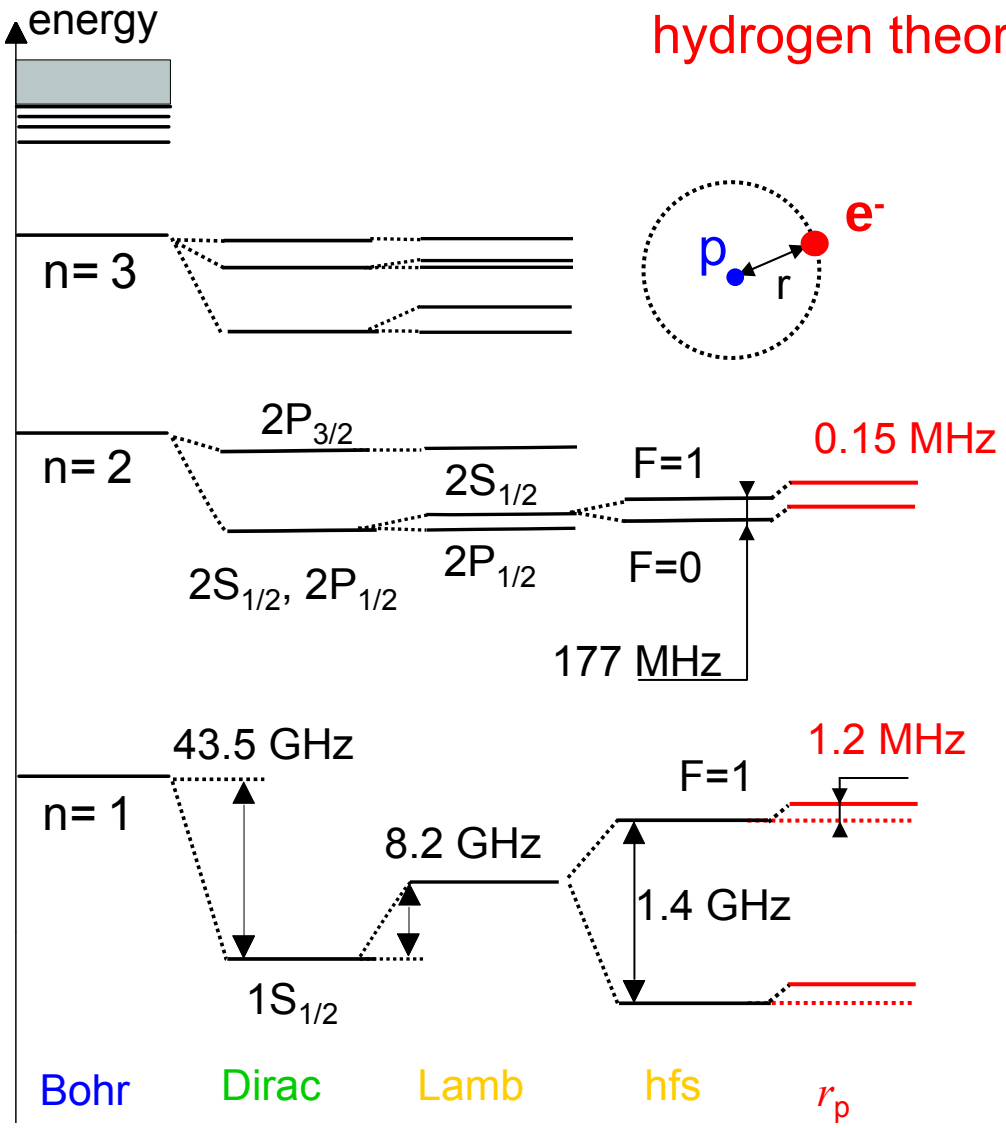

$$\vec{q} = \vec{p_f} - \vec{p_i} \qquad q = 2p\sin\left(\frac{\theta}{2}\right)$$

$$\frac{d\sigma}{d\Omega}(E_i,\theta) \approx \underbrace{\left[\frac{d\sigma}{d\Omega}(E_i,\theta)\right]}_{\text{Rutherford}} G(q^2)$$

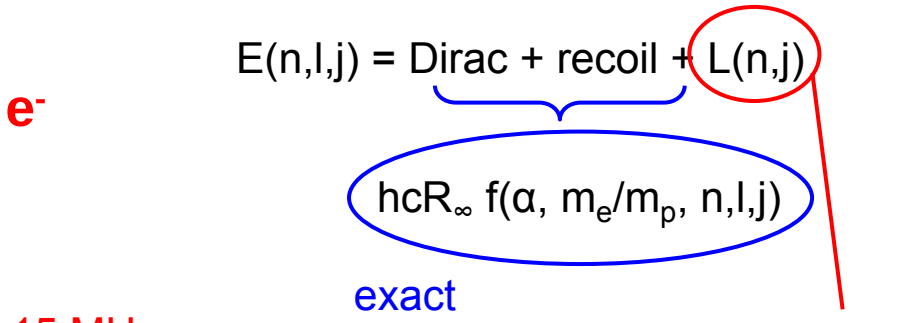
$$G(q^2) = \int d^3r\, e^{iqr}\frac{\rho(r)}{4\pi} \approx 1 - \frac{r_p^2}{6}q^2$$

Limiting factor

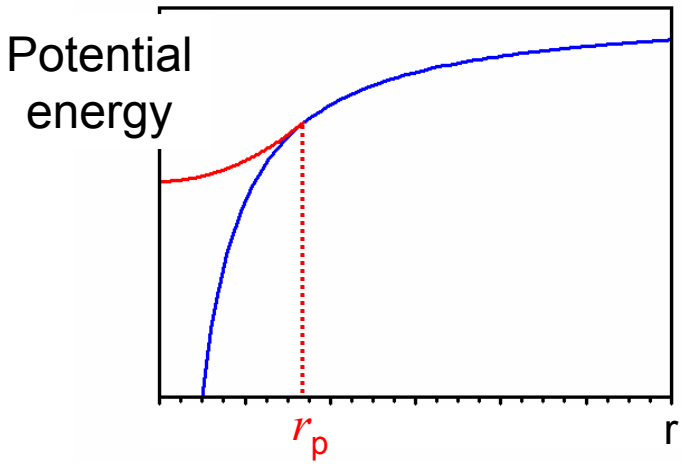
# hydrogen theory




$$E_n = -\frac{R_\infty hc}{n^2}$$




- QED corrections ($1/n^3$)
- relativistic recoil
- charge radius of the proton ($1/n^3$)




$E(1S_{1/2})$= -3 288 086 857 127.6 (3.1) kHz.

# hydrogen spectroscopy experiments

$$E(n,l,j) = hcR_\infty\, f(,\alpha, m_e/m_p, n,l,j) + \text{recoil} + L(n,j,r_p) \approx \frac{R_\infty}{n^2} + L(n,r_p)$$

MPQ Garching $\quad \nu(1S-2S) = (1-\frac{1}{4})R_\infty + L(1S) - L(2S)$

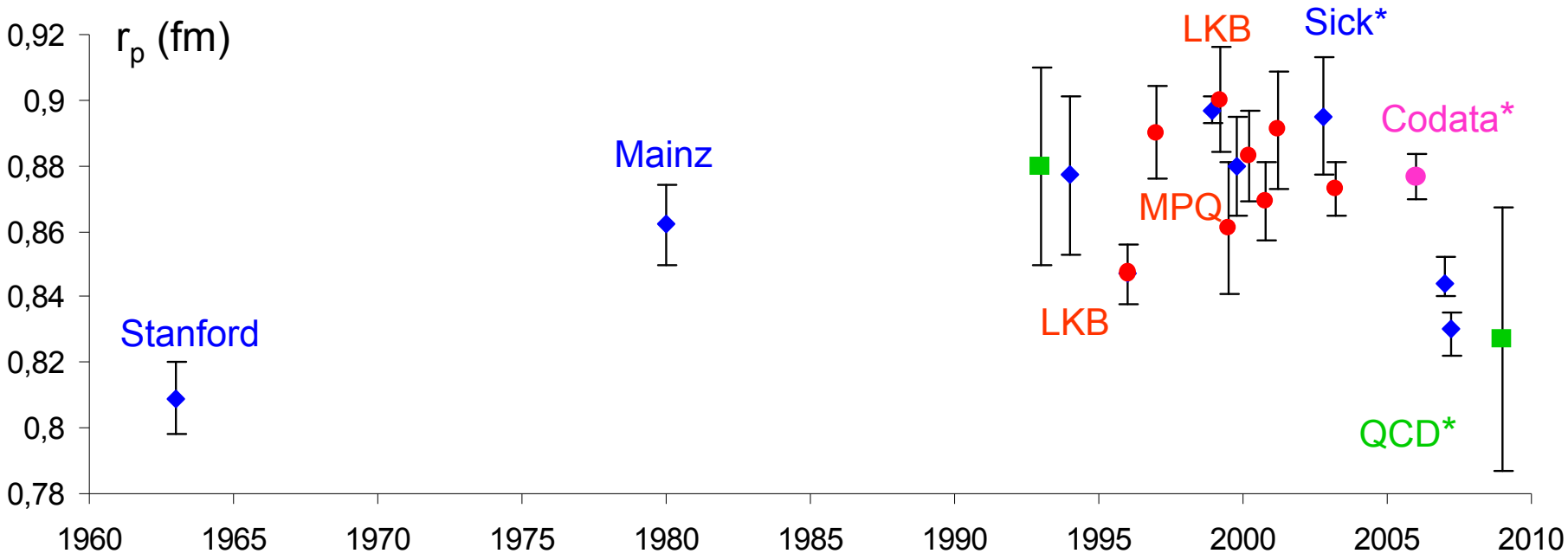
LKB Paris $\quad \nu(2S-8S) = (\frac{1}{4}-\frac{1}{64})R_\infty + L(2S) - L(8S)$

S. Karshenboim
K. Pachucki $\quad L(1S) - 8L(2S) = \text{precisely calculated}$

Linear combinations $\rightarrow$ $R_\infty$, $L^{exp}(1S)$

$L^{exp}(1S) = 8172.840\ (19)$ kHz + QED $\rightarrow$ $r_p$ (1%)

# Previous determinations of the “proton radius”



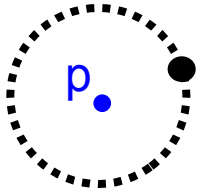

* Hydrogen data(1S-2S and 2S-nS/nD) + QED re-evaluation : 0,8760 (78) fm

* Electron scattering (all meas+re-analysis): 0,895 (18) fm

* QCD (lattice cal.) : 0,827 (40) fm

# Muonic hydrogen spectroscopy

Exotic atom

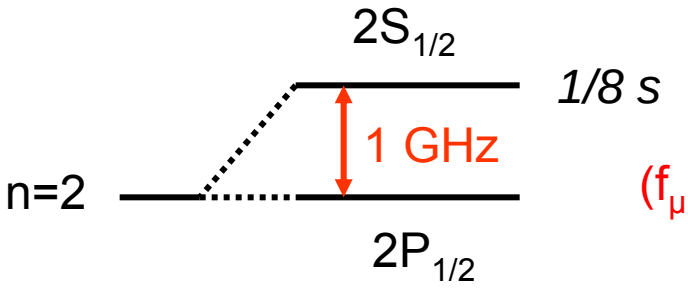



$\frac{m_\mu}{m_e} = 207$

radius : ~ $a_0/200$

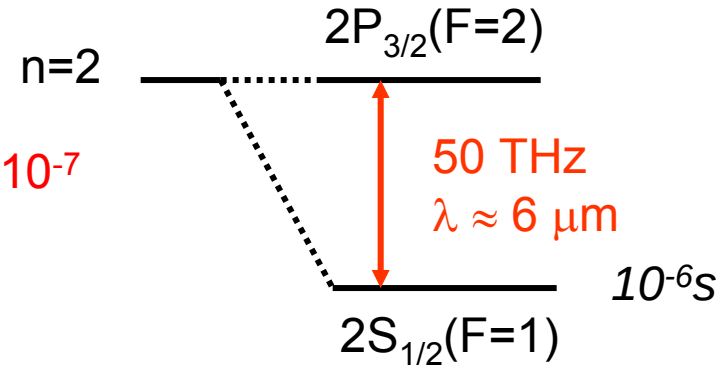
$$E_{NS} = \frac{4}{3}\left(\frac{m_r}{m_e}\right)^3 \frac{R_\infty hc}{n^3}\left(\frac{r_p}{a_0}\right)^2 \delta_{l0}$$

electronic hydrogen : e-p




$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \approx 10^{-7}$

muonic hydrogen : μ-p




$\Delta E_{2S\text{-}2P}$= 50.7724(12) – 1.2637 $r_p^2$ – 0.0084 $r_p^3$ = 49.8083 (149)THz

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   - proton properties
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   - analysis
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   - signal
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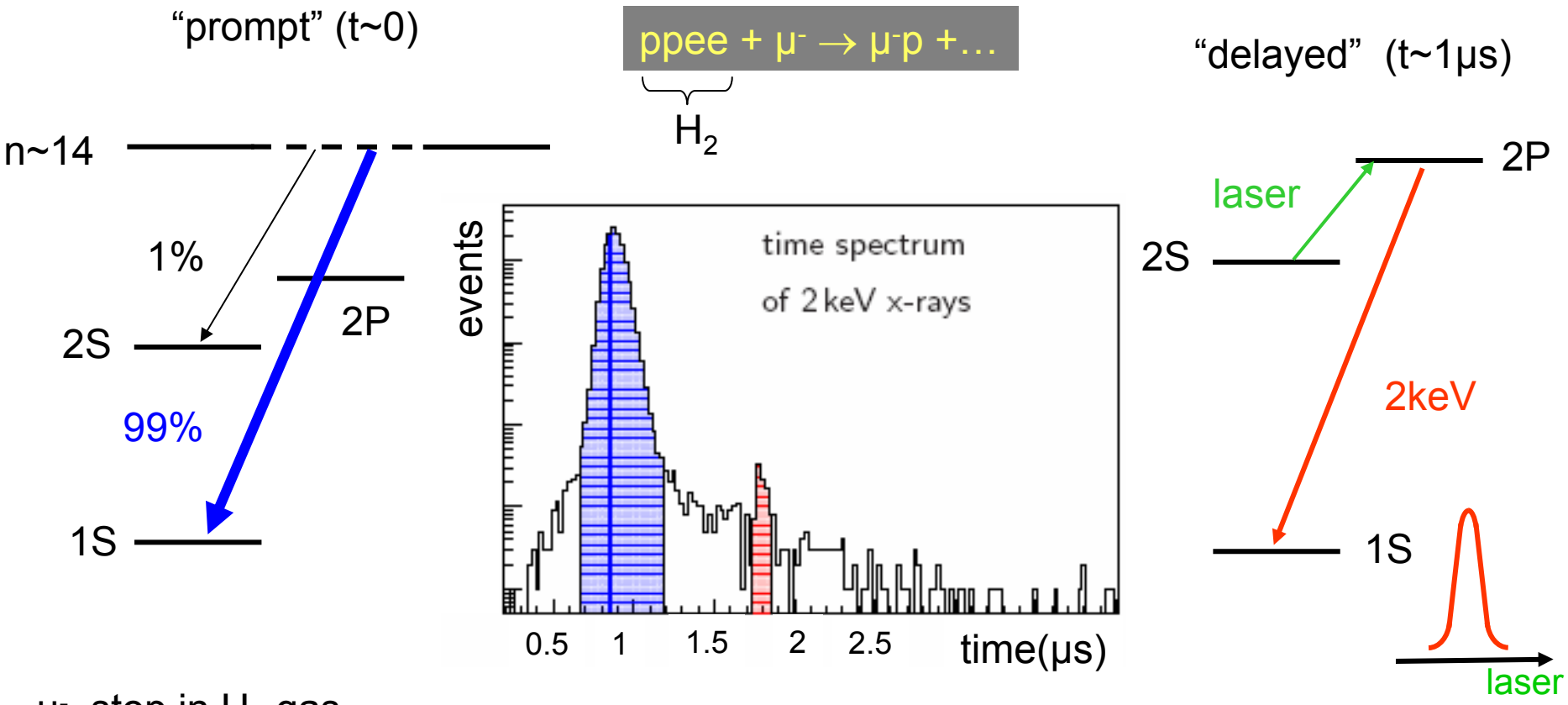
   - outcomes
4. Future

# Principle of the experiment




$\mu^-$ stop in $H_2$ gas
$\Rightarrow$ $\mu p^*$ atoms formed (n~14)

99%: cascade to 1S emitting prompt K$\alpha$,K$\beta$,…

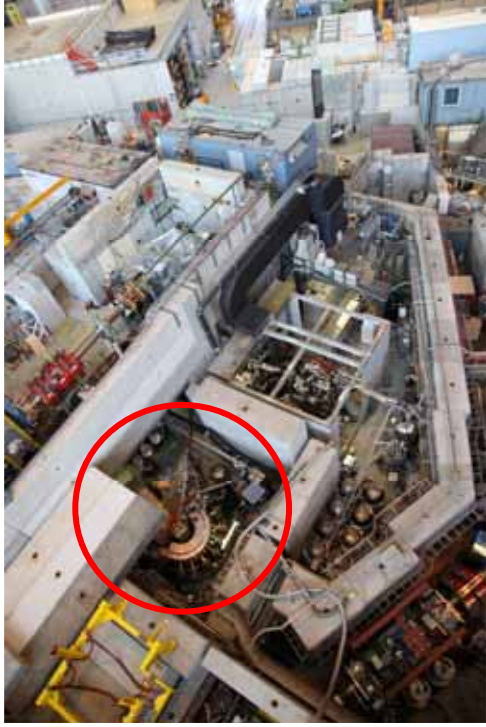
1%: long lived 2S state ($\tau$ ~ 1μs at 1mbar)

Fire laser ($\lambda$~6μm, $\Delta$E~0.2eV)
$\Rightarrow$ induce μp(2S-2P)

$\Rightarrow$ observe delayed K$\alpha$ x-rays

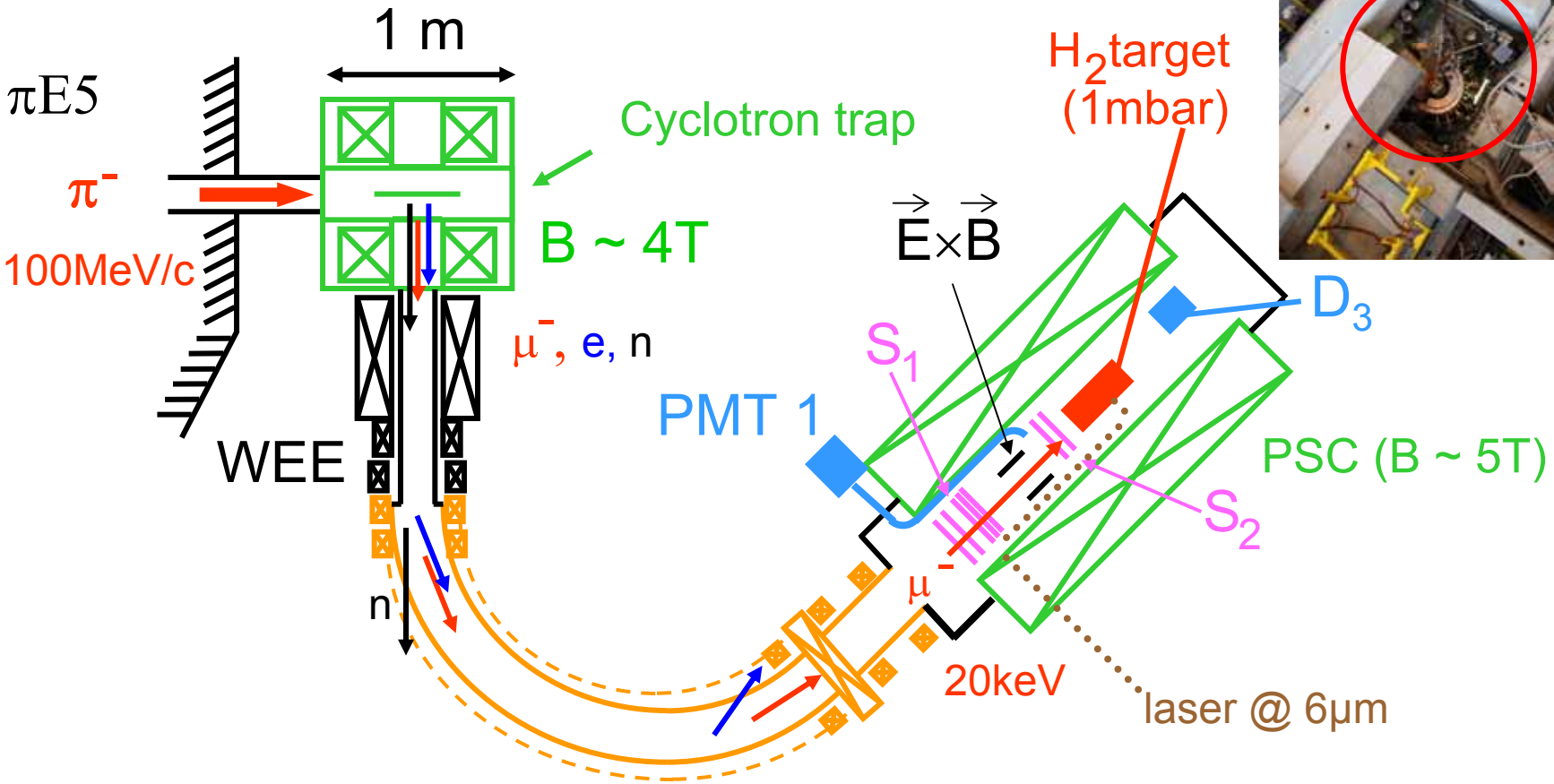
$\Rightarrow$ normalize $\frac{\text{delayed K}\alpha}{\text{prompt K}\alpha}$ x-rays

# muonic hydrogen source



p (500Mev, 2mA) → carbon target →

$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ (300 $\mu^-$/s)




$S_1$ $S_2$ → T.O.F. → laser trigger

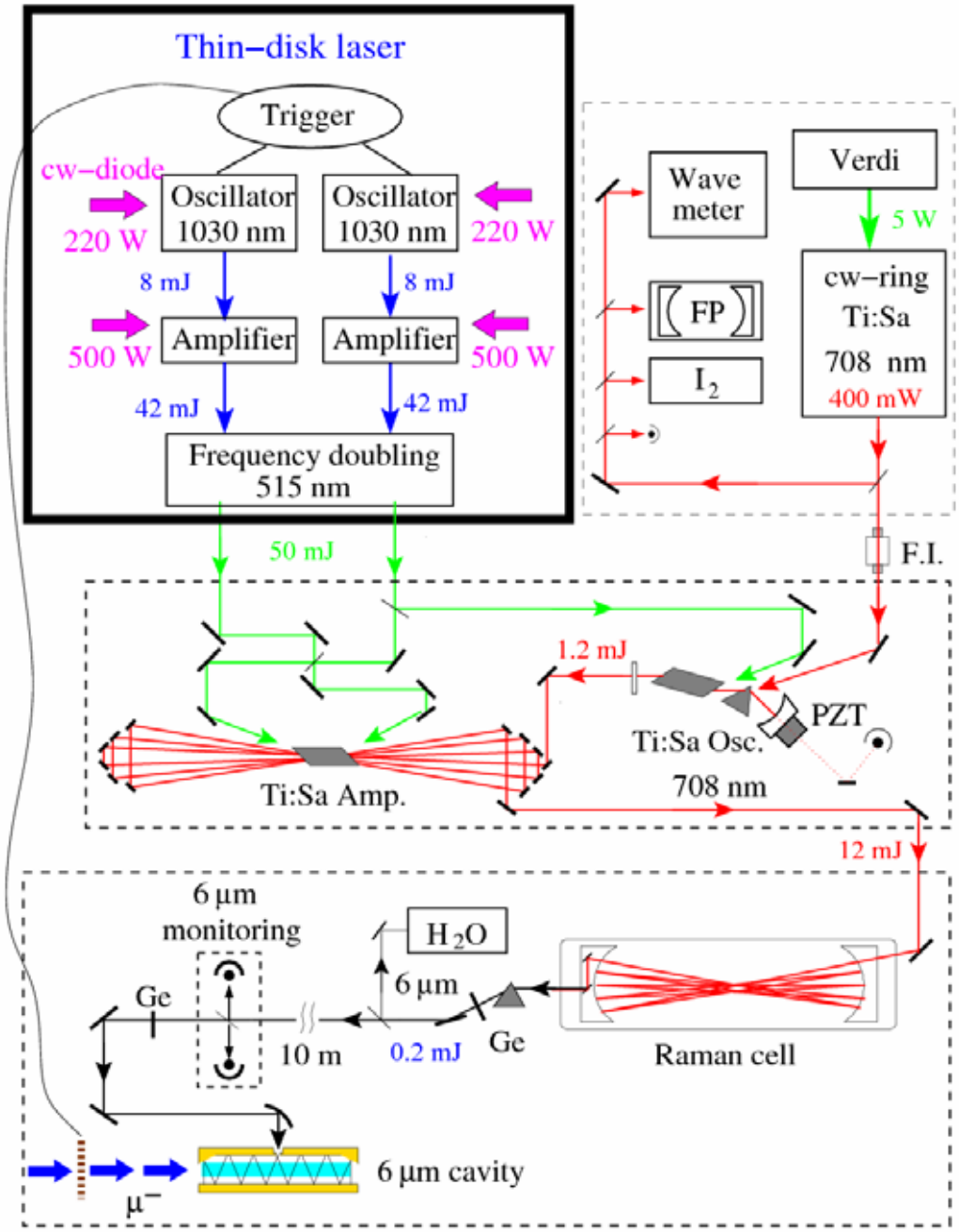
# muon life in CREMA experiment

pion $\pi^-$

muon $\mu^-$

photon from scintillator

# Laser chain




- Each single muon triggers the laser system (random trigger)
- 2S lifetime ~1µs → short laser delay (disk laser)
- 6 µm tunable laser pulse (0.2mJ)

µ⁻ trigger

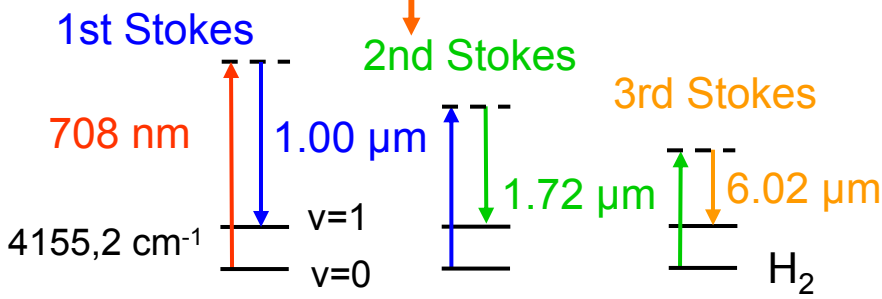
↓

Thin disk laser (1030 nm) + LBO (515 nm)

↓

pulsed TiSa oscillator + amplifier
(708 nm determined by cw TiSa seeding)

↓




↓

Multipass cavity at 6µm
surrounding the H2 target

# X-rays analysis → noise rejection

Example : FP 900 - 11 hrs meas.

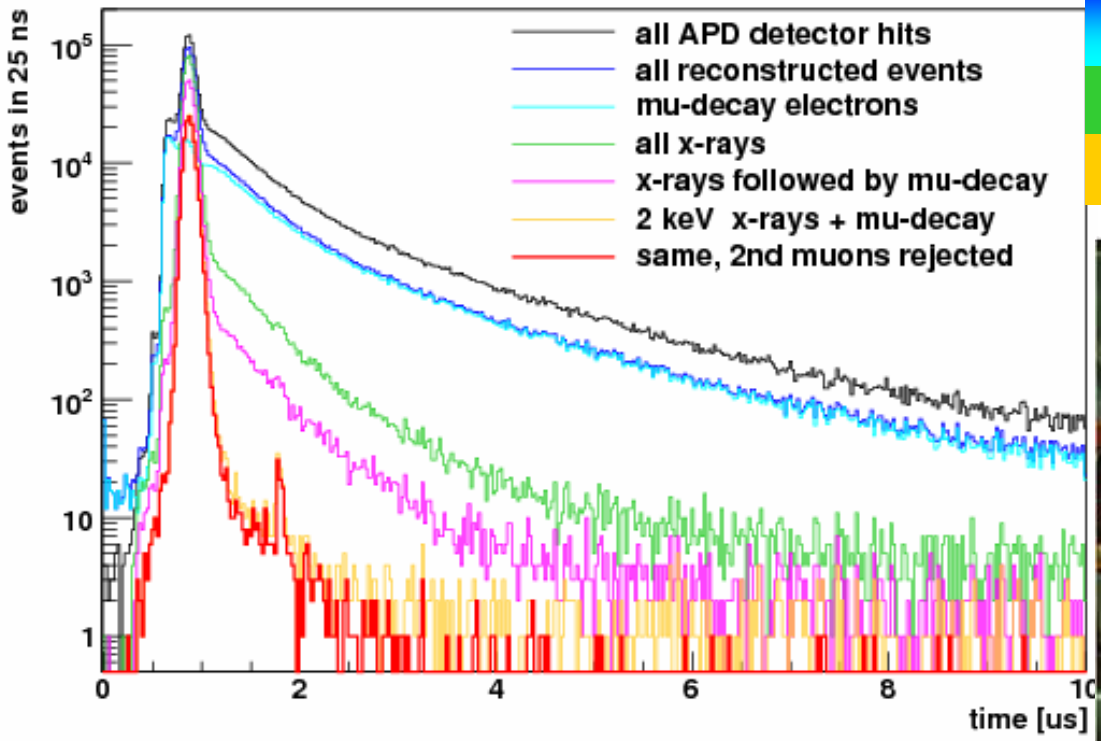
- 400 $\mu^-$/s
- 240 laser shot/s
- 860 000 laser shot/hour
- 1.56 million detector clicks
- 19600 clicks in the laser region
- expected 2-3 laser induced events/hour !

time signature in LAAPD

- photon < 10keV → 1 shot in the LAAPD
- $e^-$ in B = 5T → many counts in detectors

energy signature in LAAPD

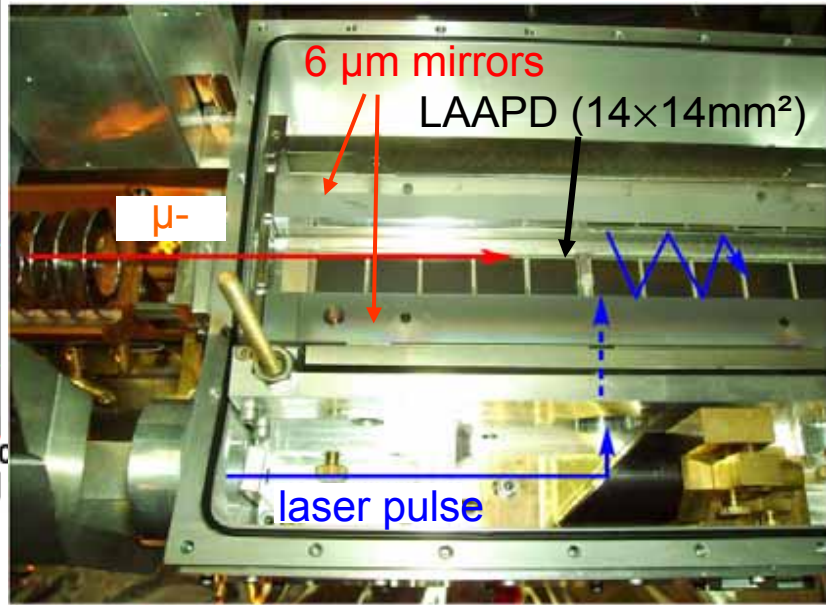
- E > 8keV ⇔ electron
- 1keV < E < 8keV ⇔ X ray
- E<1keV ⇔ neutron




$\mu \rightarrow e\ \nu_\mu\ \bar{\nu}_e$

x rays multiplicity 1

LAAPD energy resolution



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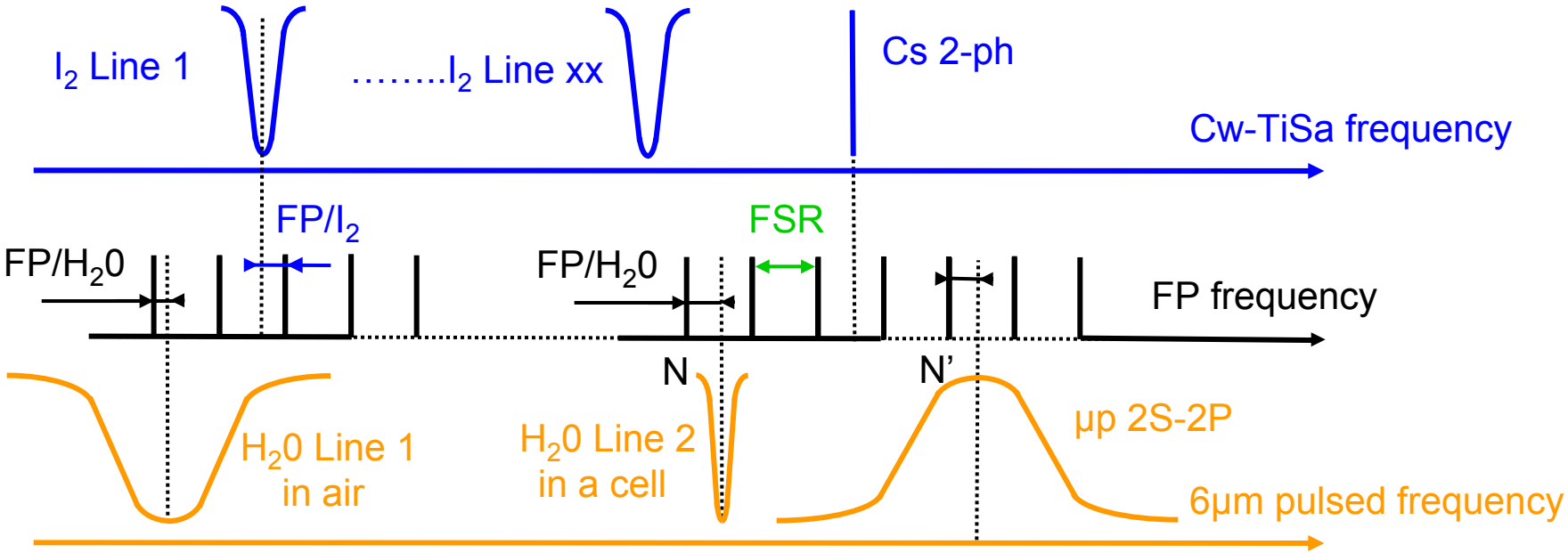
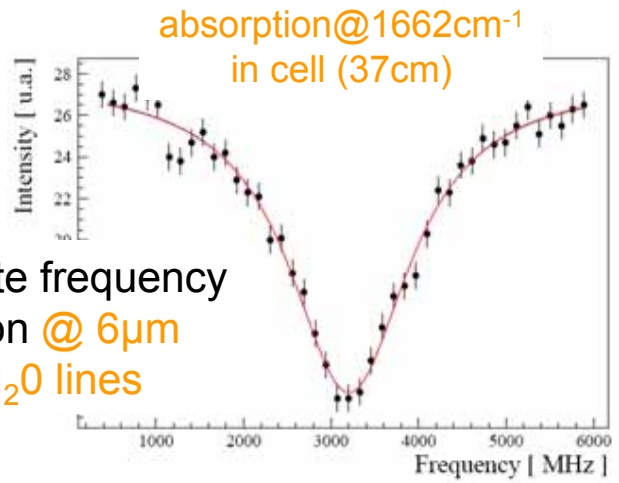
4. Future

# Laser chain : frequency calibration

FSR measured/controlled in cw with $I_2$ (1 ph abs), Cs (2 ph fluo), Rb (2 ph fluo), lines




FP absolute frequency calibration @ 6μm with $H_20$ lines

$\nu(\mu p{:}2S\text{-}2P) = \nu(H_20 \text{ Line } 2) + (N\text{-}N')\ FSR$

# μp ($2S_{1/2}(F=1)$-$2P_{3/2}(F=2)$) : uncertainty budget

## Statistics

| | |
|---|---|
| • uncertainty on position (fit) | 700 MHz (~ 4 % of $\Gamma_{nat}$) |

$\Delta\nu_{experimental}$= 20 (1) GHz ( $\Gamma_{nat}$ = 18.6 GHz )

## Sources :

| | |
|---|---|
| • Laser frequency ($H_20$ calibration) | 300 MHz |
| • AC and DC stark shift | < 1 MHz |
| • Zeeman shift ( 5 Telsa) | < 30 MHz |
| • Doppler shift | < 1 MHz |
| • Collisional shift | 2 MHz |
| TOTAL UNCERTAINTY ON FREQUENCY | 762 MHz |

## Broadening :

| | |
|---|---|
| • 6 μm laser line width | ~ 2 GHz |
| • Doppler Broadening | < 1 GHz |
| • Collisional broadening | 2.4 MHz |

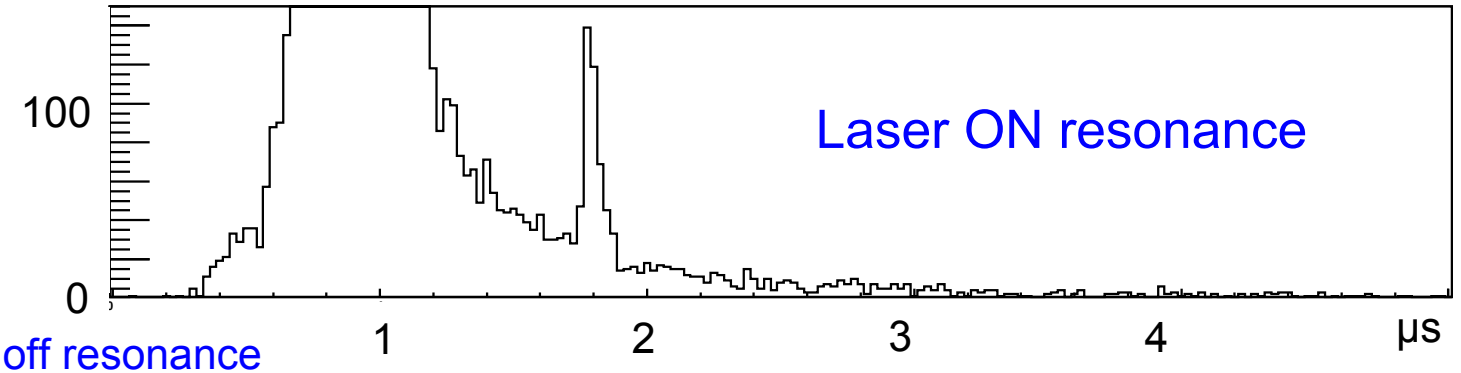
$\nu$ (μp : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.88 (76) GHz ($1.6\times10^{-5}$)

# Outline

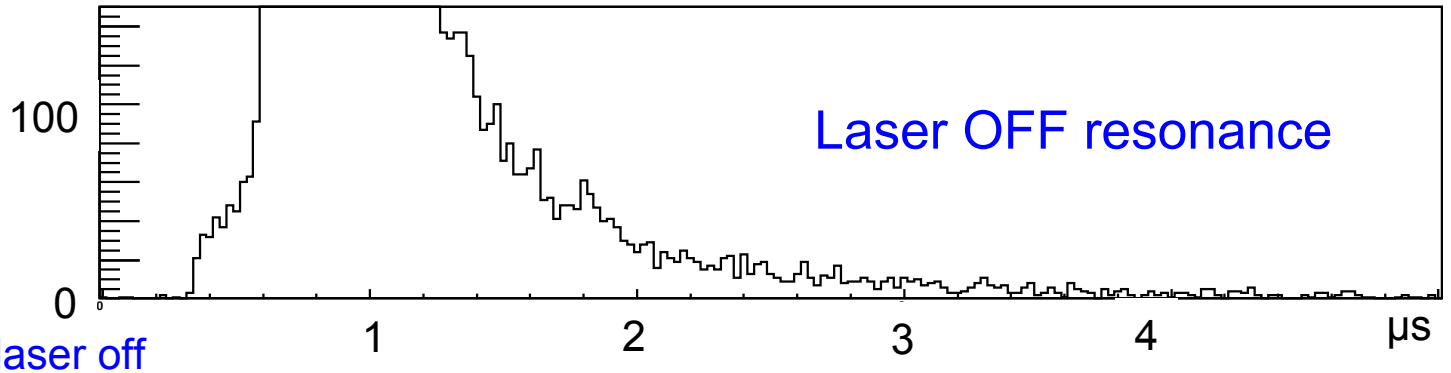
1. Introduction
   - proton properties
   - experimental proton size determinations
   - muonic hydrogen
2. Experiment
   - muonic hydrogen source
   - laser system
   - analysis
3. Results
   - signal
   - proton radius from muonic hydrogen spectroscopy
4. Future

# muonic hydrogen: $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$ time spectra

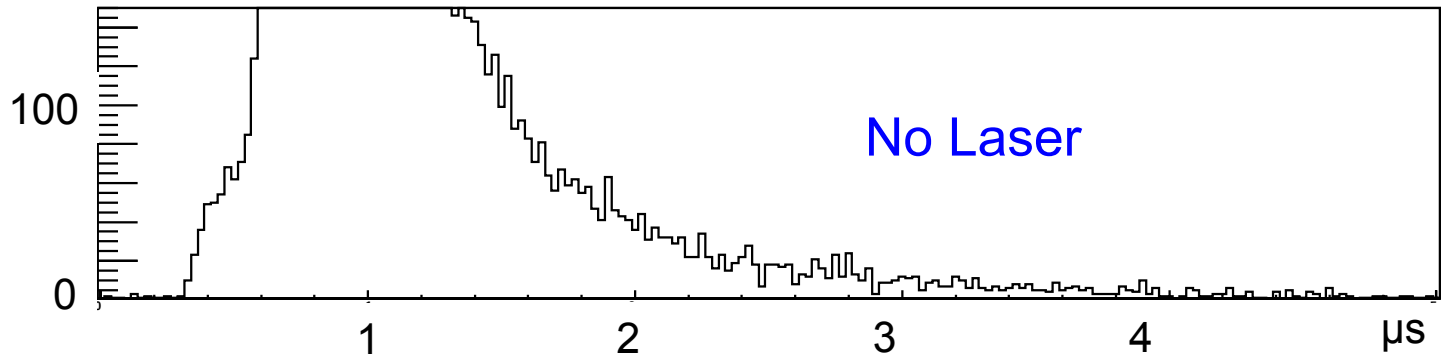
summed on resonance




summed off resonance




summed laser off



# New $r_p$ ($\mu p$ ($2S_{1/2}(F=1)$ - $2P_{5/2}(F=2)$)) : outcomes

## Muonic hydrogen Lamb shift

- check on line consistency of all publications (P. Indelicato, E. O. Le Bigot, F. Kottmann)
- independent QED calculations (P.Indelicato, P.J.Mohr) (large contributions : OK)
- check others theoreticians (PSAS 2010…)

## Hydrogen spectroscopy, Rydberg constant $R_\infty$ :

$H : \nu_{1S\text{-}2S} = ¾\ R_\infty + L(QED, r_p)$

(34 Hz) $u_r(R_\infty)$ : 2S-nS/nD (n=8,12) (~7 kHz)

$cR_\infty$ = 3 289 841 960 360.9 (21.9) kHz ($6.6\times10^{-12}$)

$cR_\infty$ = 3 289 841 960 250.9 (4.8) kHz ($1.5\times10^{-12}$)

but ~ 5 $\sigma$ shift ($3.3\times10^{-11}$)

Without Paris $cR_\infty$ = 3 289 841 960 379.8 (48.0) kHz ($14.6\times10^{-12}$)

(from Codata98 Rev. Mod. Phys. 72, n°2, April 2000)

Codata 2006 *"The experimental and theoretical H and D data remain highly consistent."*

First measurement of 1S-3S (LKB) : 2 922 743 278 678(13) kHz. EPJ D **60** p.243 (2010)

Prediction : 2 922 743 278 671.6(1.4) kHz PRL **95**, 163003 (2005)

## "New physics":

*"We consider exotic particles that, among leptons, couple preferentially to muons and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1, and spin-2 particles as an explanation."* PRL **106**, 153001 (2011)

Example of constrain : magnetic moment anomaly of the electron (Gabrielse, Kinoshita) g-2=f( $\alpha/\pi$)

With recoil (LKB) $v_r = \frac{\hbar k}{m(Rb)}$ $\alpha^2 = \frac{2R_\infty}{c}\frac{m(Rb)}{m_p}\frac{m_p}{m_e}\frac{h}{m(Rb)}$ PRL **106**, 080801 (2011)

# Prospect : muonic helium spectroscopy at PSI

2013….→ muonic helium spectroscopy

- $\mu He^+$ spectroscopy + $He^+$ spectroscopy → QED test ($Z\alpha$)
- test of the spectroscopy of muonic system

# Up date comparison

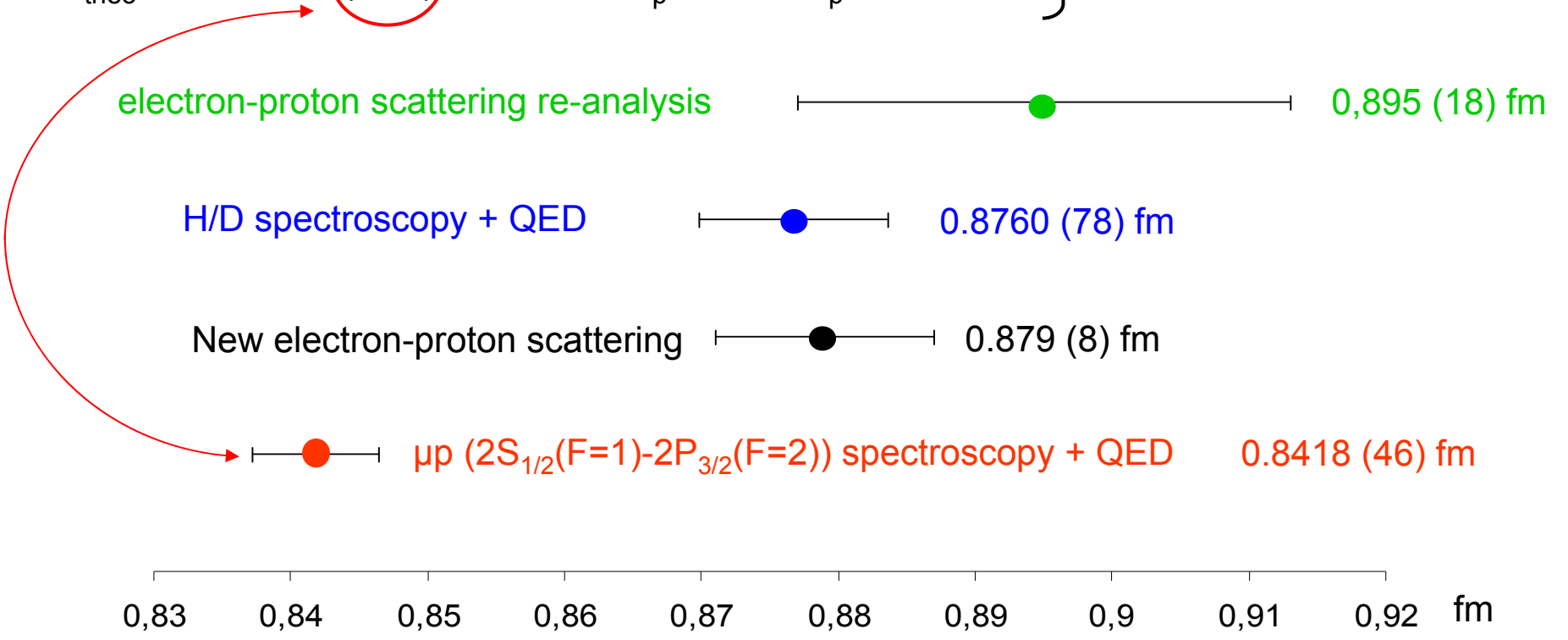
New electron-proton scattering : Phys Rev Lett 105, 242001 (2010)

Model independent analysis of proton structure…:Phys Rev Lett 107, 160402 (2011)

$\nu$ ($\mu$p : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.88 (76) GHz

$\nu_{theo}$= 50772.43(967) – 1263.69 $r_p^2$ + 8.39 $r_p^3$ GHz

$r_p$= 0.84184(36)(458) fm

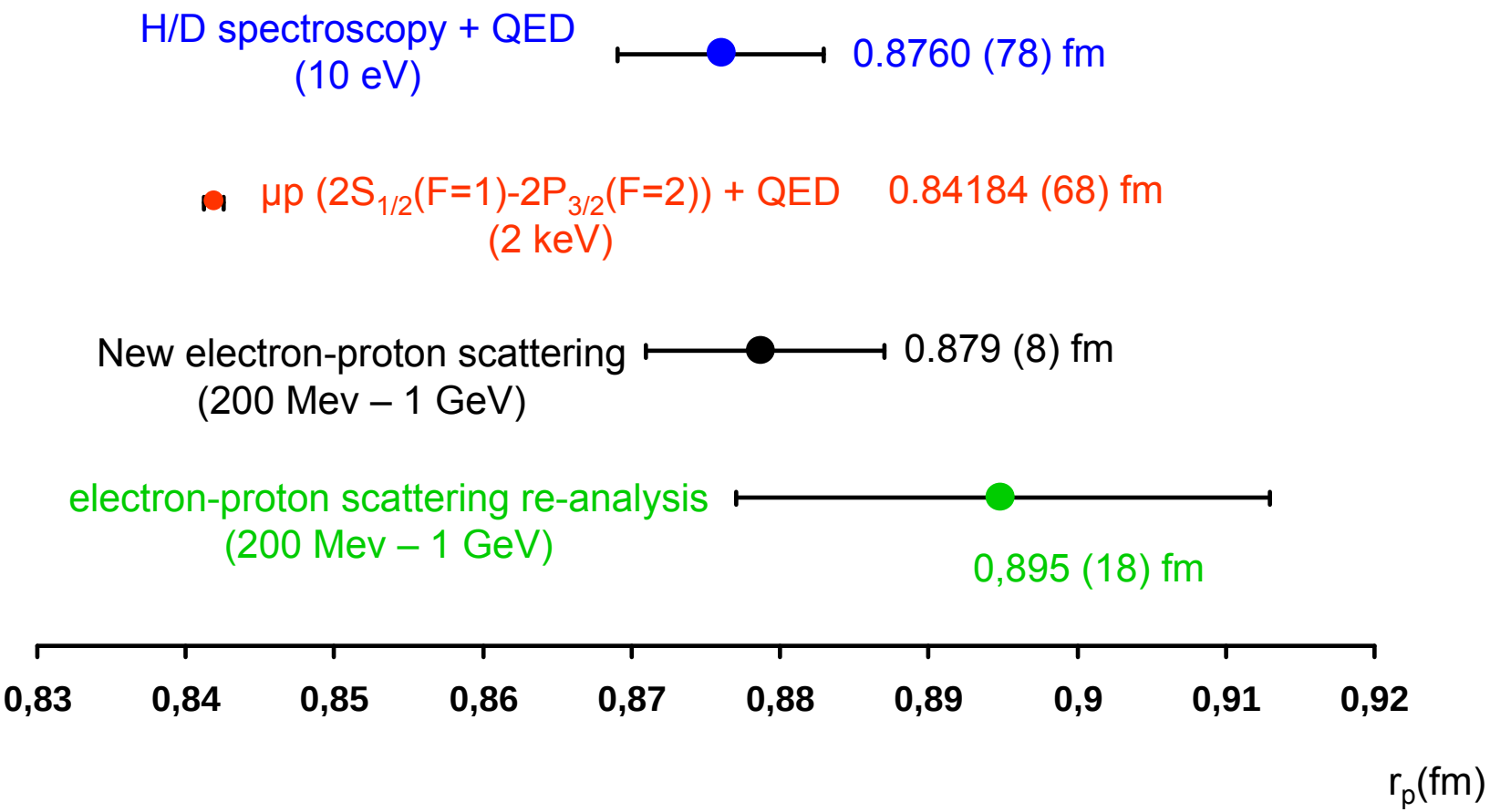


# Comparison with other best determinations

$\nu_{theo}$= f($R_\infty$, QED, $r_p$) =50772.43(1.18) – 1263.69 $r_p^2$ + 8.39 $r_p^3$ GHz

$\nu$ (μp : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.88 (76) GHz

$\Rightarrow r_p$= 0.84184(36)(56) fm




**New electron-proton scattering : Phys Rev Lett 105, 242001 (2010)**

**Muonic hydrogen : *Nature* 466, 213-216 (2010)**